Feints

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General Idea

"Always mystify, mislead and surprise the enemy, if possible"

-General Thomas J. "Stonewall" Jackson

Operation Overlord

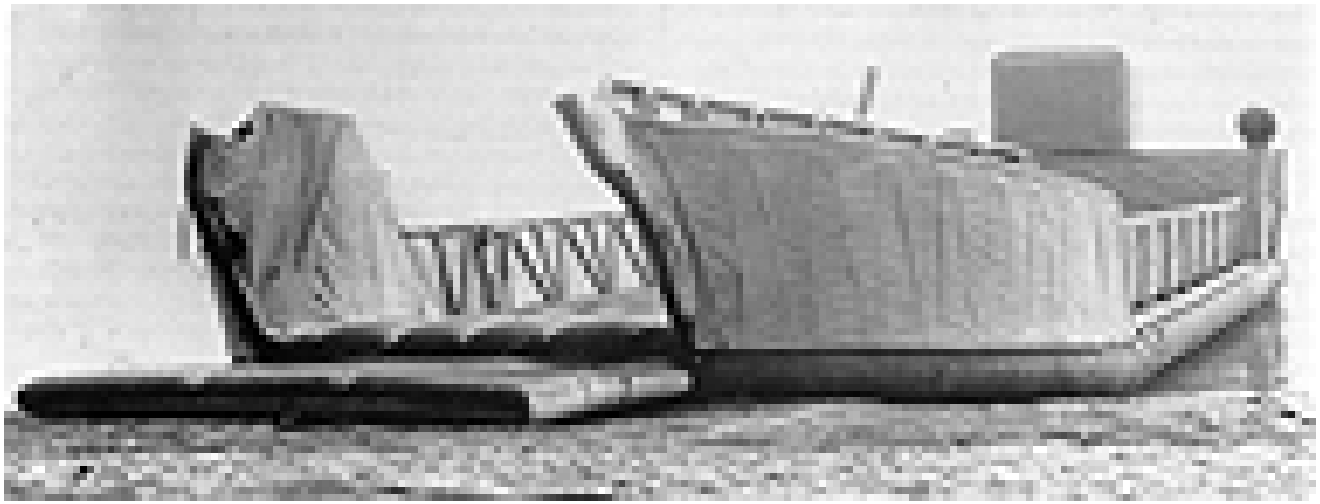
- Allied Invasion of France
- AKA Operation Neptune
- Dover to Calais natural spot
  - Minimized supply lines, extensive roads available
- Obvious to Germans as well
- Allies planned a feint: Operation Fortitude

Operation Fortitude

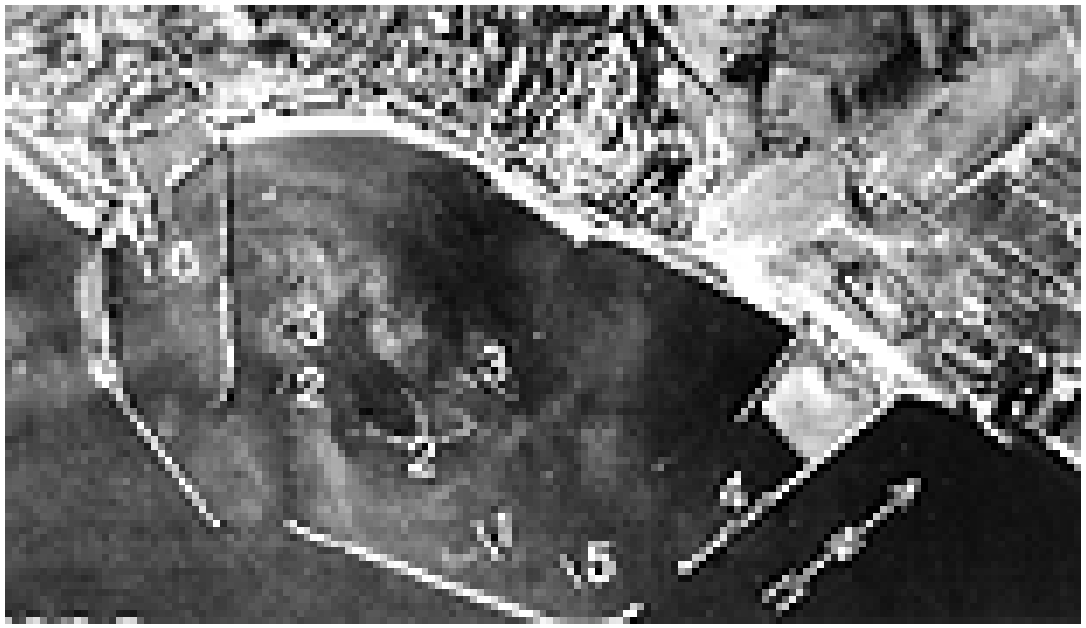
- ✯ Fool the enemy into believing in a Calais attack
- ✯ Mythical US 1st Army Group, led by George Patton
- ✯ Plywood aircraft, rubber landing craft, inflatable tanks
- ✯ Fake coded transmissions, Real maneuvers
- ✯ More bombs dropped on Calais than on Normandy
- ✯ Dummy paratroopers that smoked and played recorded gunfire noises.
- ✯ Fake armada of radar-jamming boats
- ✯ Planes dropped aluminum foil to make radar “light up” and be unusable

Inflatable Tank

Dummy Ship



German Aerial Photo of Dummy Ships



Operation Fortitude

- ✯Wildly Successful
- ✯Germans held 19 divisions at Calais
  - Waiting for an attack that never came
  - Believed that Normandy was the feint

What is a Feint?

A feint is an action conducted for the purpose of deceiving an adversary as to the time or location of the main offensive action.

Requires imperfect signaling technology

Contrast to Signaling

- e.g. Milgrom Roberts, Spence
- Convince receiver that one is the “good” type
- In contrast, a feinter wants to convince receiver that one is the *other* type
- Were the allies invading at Pas de Calais, goal would be to convince Germans that the invasion was coming at Normandy

Fool All of the People Some of the Time

- In signaling, one type ignores the signaling effect
- Separating: bad type maximizes
- Pooling: includes equilibrium where good type maximizes
- In contrast, in feinting, both types invest in misleading

Other Examples

"Create havoc in the west and strike in the east"

-Sun Tzu

- Concealing the Tidewater Pipeline from Standard Oil Trust
- Concealing successful exploration from competitors
- Irrelevant PCS Auction Bids by GTE
- Irrelevant questions in depositions
- Mazda Miata unveiled as a very different car
- Naked reverse in football

Major Results

With a Noisy Signal:

- Feinter employs pure strategy investment
- Receiver also employs pure strategy
- Signal causes belief updating
- Feinter prefers noisier signal

Major Results, continued

With a Revealing Signal:

- Feinter mixes
- Sometimes attacks with a weak force
- Signal may be uninformative
  - Receiver still reacts partially to signal
- Feinter prefers less noisy signal

Attacker Model

- Two locations, *a* and *b*
- Attacker allocates force *x* to *a*
- 1-*x* allocated to *b*
- A signal *S* in$\{\alpha,\beta\}$ is generated to defender
- Defender chooses *y* allocated to *a*, 1-*y* to *b*
- Prob($S=\alpha|x$)=$p(x)$

Payoffs

- Defender allocates y_S to a, 1-y_S to b.
- Defender obtains $qU(y)+(1-q)U(1-y)$
- y determined by inference about q
- Payoffs to attacker is

$$\pi = q[x - p(x)\, y_\alpha - (1-p(x))\, y_\beta]$$
$$+ (1-q)[1- x - p(x)\, (1-y_\alpha)-(1-p(x))\, (1-y_\beta)]$$
$$= 1-q + (2q-1)[x - p(x)\, y_\alpha - (1-p(x))\, y_\beta]$$

- Attacker would like to choose x=1 if q > ½, otherwise x=0.
- This would provide information to the defender about q
- The desire to mislead creates the possibility of a feint

Inference

Attacker knows q

Defender draws inference about q given signals:

$$E\{q \mid S = \alpha\} = \frac{\int qEp(X(q))f(q)dq}{\int Ep(X(q))f(q)dq}$$

$$E\{q \mid S = \beta\} = \frac{\int q(1-Ep(X(q)))f(q)dq}{\int (1-Ep(X(q)))f(q)dq}$$

Assumptions on p

- The probability p
- is increasing and differentiable,
- satisfies $p(z)+p(1-z)=1$
- satisfies $p(0)=0$.
- A technology p_1 is more revealing than p_2 if
  $|p_1(x)- ½| \geq |p_2(x) - ½|$.
- Technology is “noisy” if less revealing than identity, and “revealing” if more revealing than identity.

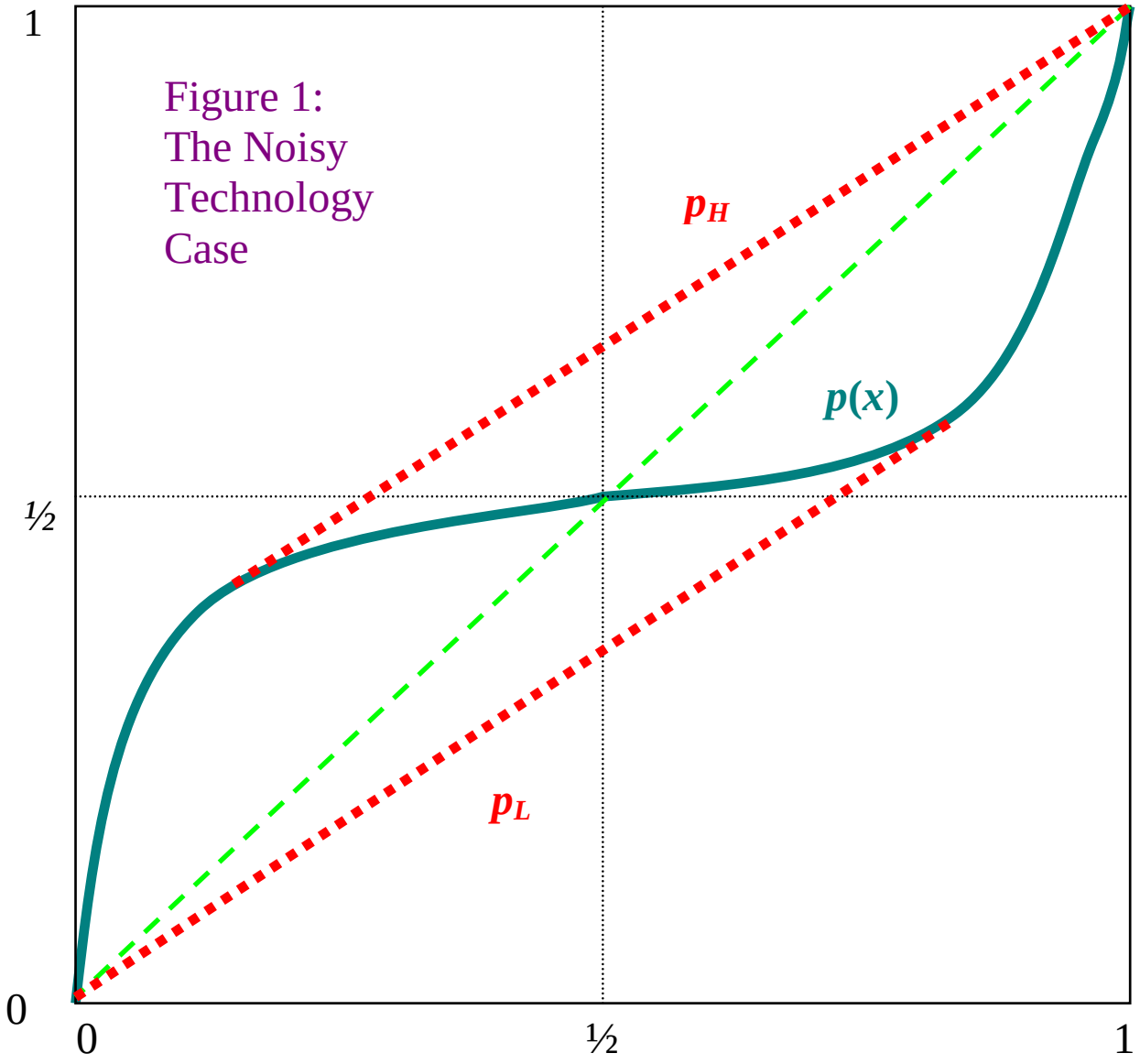


Figure 1:
The Noisy
Technology
Case

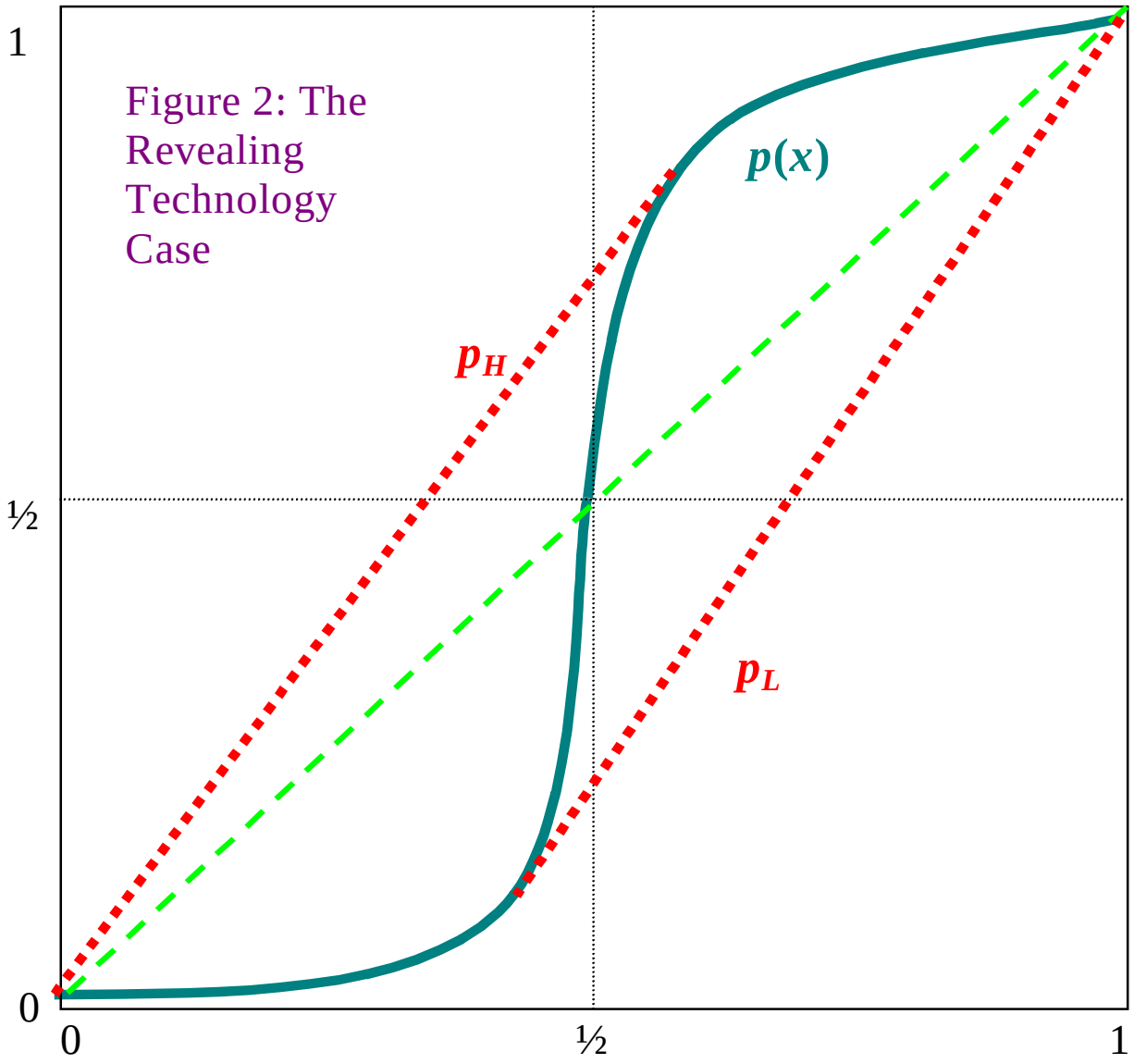


Figure 2: The Revealing Technology Case
1
½
0
0
½
1
$p(x)$
p_H
p_L

Defender

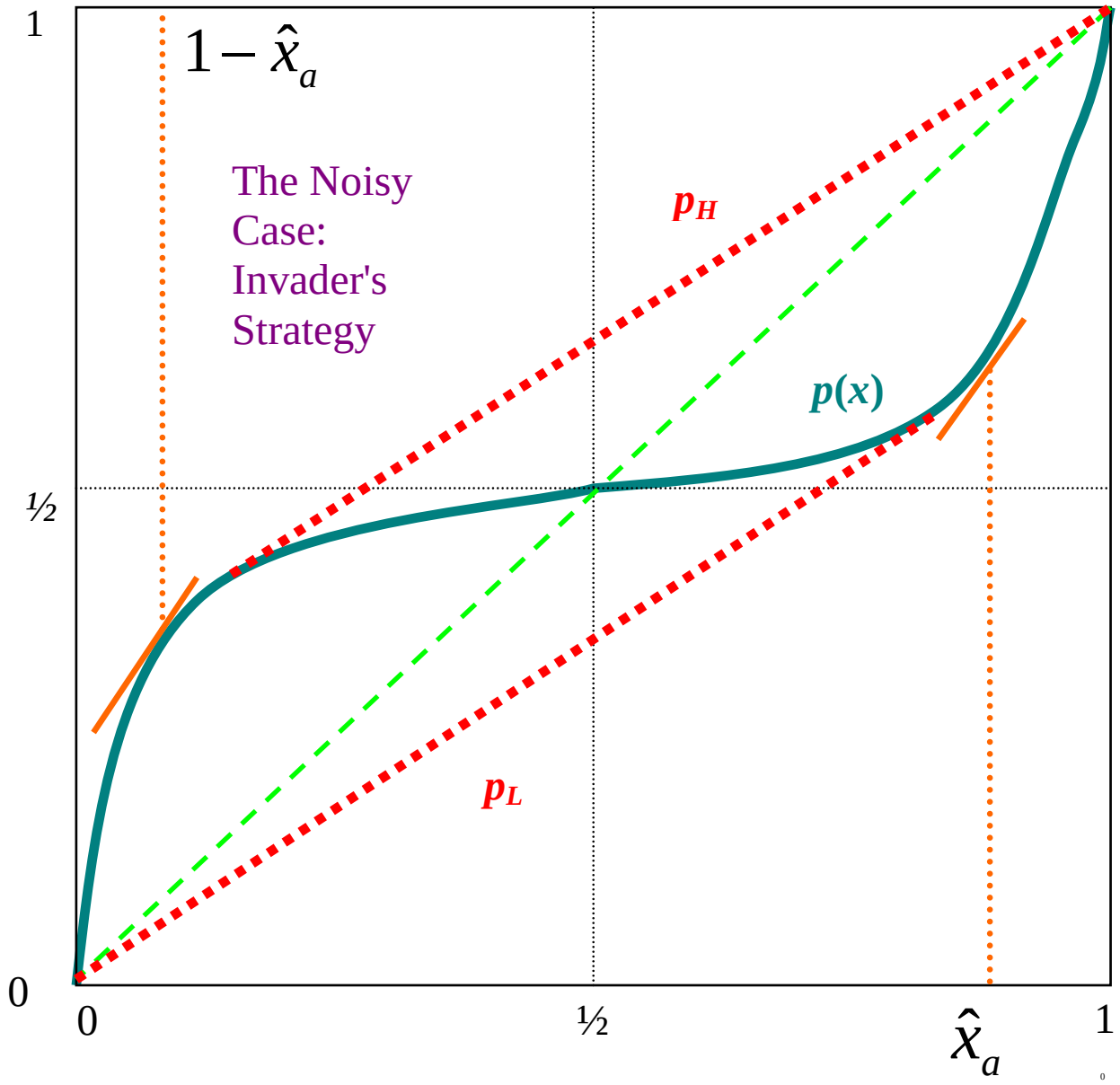
- Defender optimization

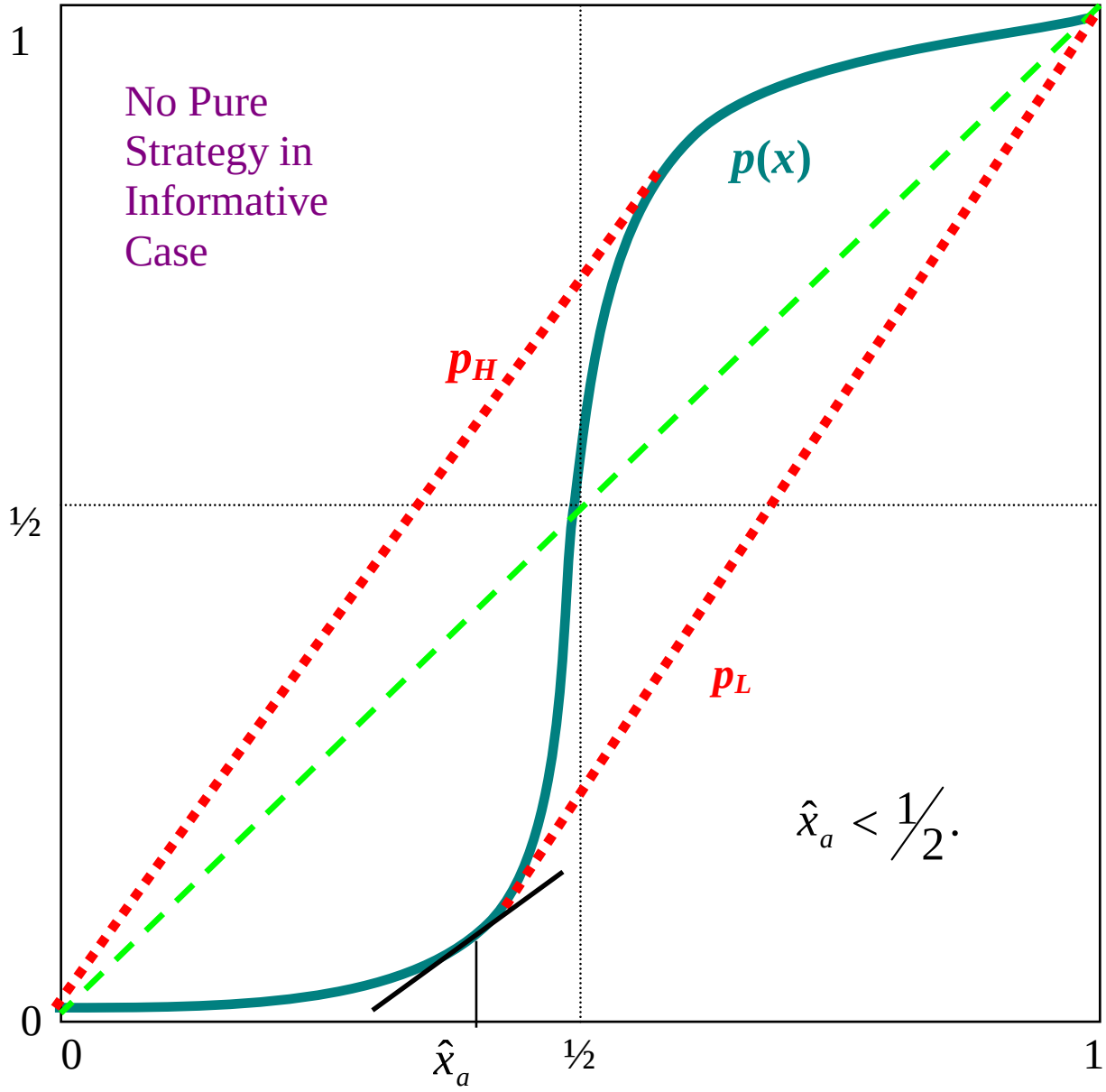
$$\frac{U'(y_s)}{U'(1-y_s)} = \frac{1-E\{q|s\}}{E\{q|s\}}$$

Pure Strategy Equilibria

- A pure strategy equilibrium is an increasing $x(q)$.

1. Suppose the signaling technology is noisy. Then there is a unique equilibrium in which $\frac{1}{2} < x(q) < 1$ if $q > \frac{1}{2}$. The defender allocates more force to a if the signal is α.
2. If the signaling technology is revealing, then there is no pure strategy equilibrium


1
$1-\hat{x}_a$
The Noisy Case: Invader's Strategy
p_H
$p(x)$
p_L
½
0
0
½
1
$\hat{x}_a$


1
½
0
No Pure
Strategy in
Informative
Case
p(x)
p_H
p_L
$\hat{x}_a < 1/2.$
0
$\hat{x}_a$
½
1

Revealing Signal

- Pure strategy equilibrium doesn't exist
- Attacker must randomize
- Signal must be on the convex hull of p function
  - Otherwise preferred strategy (higher EX, same Ep) exists

Attacker Behavior

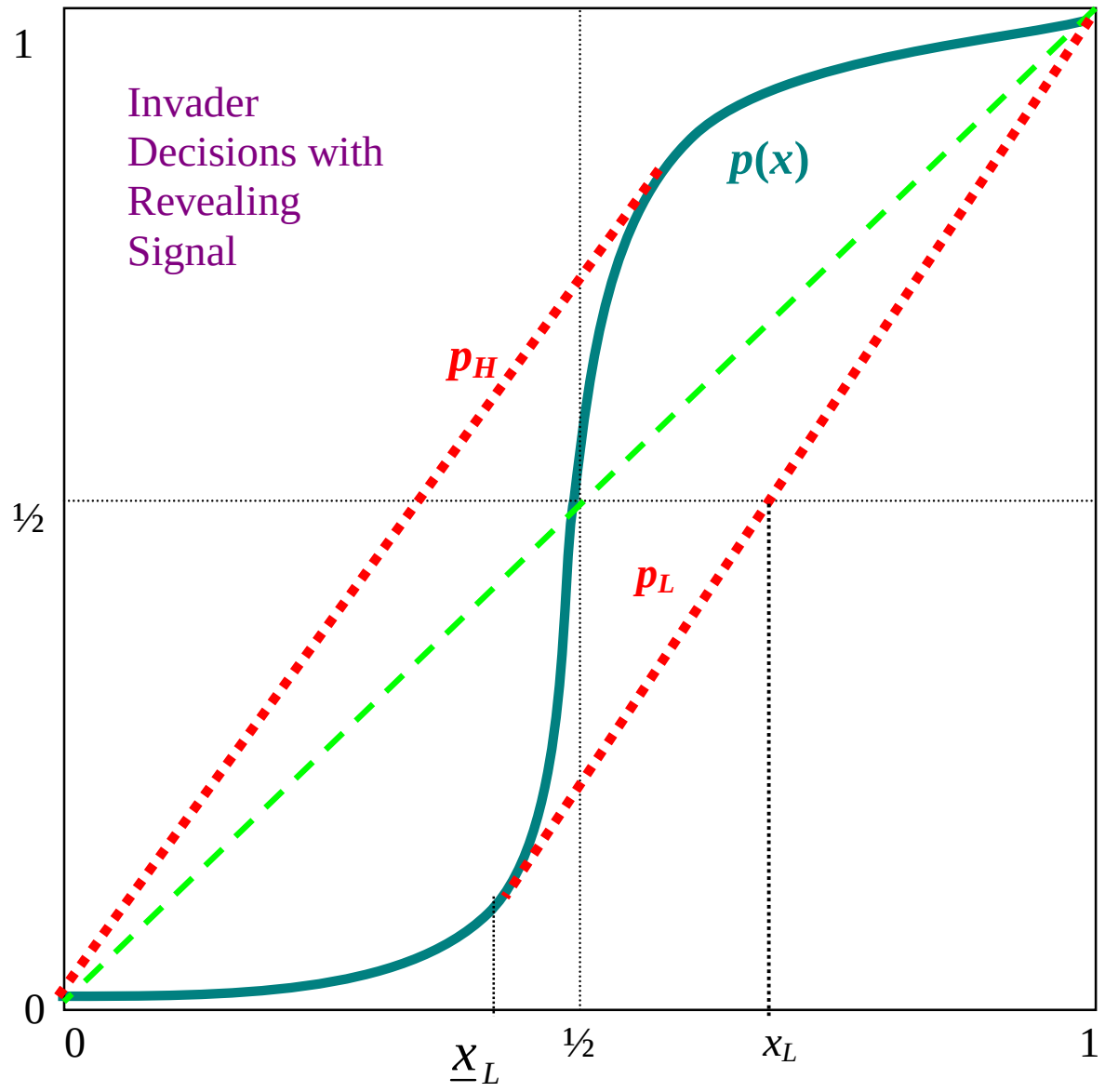
- $q > ½$, attacker uses $x_1 < ½$ with probability θ; otherwise, attacker uses 1.
- Symmetry defines $q < ½$
- x_1 is a “stealth attack”
  - Less than half force applied to most important battle

Mixed Strategy Equilibrium

Suppose the signaling technology is revealing.

Then there is a unique equilibrium in which

- the attacker randomizes over a full attack and a stealth attack,
- if defender risk neutral, signal generated is uninformative
  - defender reacts anyway


1
½
0
Invader
Decisions with
Revealing
Signal
$p(x)$
p_H
p_L
0
$\underline{x}_L$
½
x_L
1

Risk Averse Defender

- Risk aversion causes more even split in the defender's forces
- Increases attacker payoff in noisy environment
- Doesn't matter in revealing environment

Technological Choice

- Invader with noisy technology prefers noisier technology, to conceal attack
- Invader with revealing technology prefers more revealing technology
- Revealing technology makes weak force attack more profitable

Conclusions

Two main types of feints

- When signal isn’t too revealing, “small” force devoted to feinting
- When the signal is very revealing, get randomized attack force

Conclusions

When signal isn’t too revealing:

- Pure strategy in attack force
- Some effort to feinting
- Main force always attacks
- Signal generated is informative (changes beliefs)
- Attacker prefers noisier signal

Conclusions

When the signal is very revealing

- Sometimes “main” force is a diversion
- Signal generated may be uninformative
  - risk neutral defender only
- Nonetheless, defender responds to signal
- Attacker prefers more informative signal

Conclusions

Analysis robust to

- Null signals
- Different resource levels
- Continuum of signals
- Risk aversion

Signalling model may prove useful in other contexts