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Tariffying auctions

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A strategy to convert quotas to tariffs is to auction the quota rights and use the realized auction prices as guides to setting tariffs. In the 1980s, New Zealand employed auctions to allocate quota licenses. We analyze the relationship between tariff-equivalents and auction prices for auctions with resale. Using panel data from New Zealand's quota license auctions, we estimate the expected value of the tariff equivalent. We exploit data from secondary market prices to test the model's predictions. The predictions fail, suggesting either that auction prices may understate or that aftermarket prices overstate the true tariff equivalent.

1. Introduction

■ Auctions have become very popular among policy makers worldwide as a mechanism to allocate public resources. Economic theory suggests that auctions offer many advantages. Under what seem to be mild conditions, auctions ensure the efficient allocation of goods and raise substantial revenues for the public purse. They also offer a source of information that policy makers may be able to exploit in implementing other policy goals. For example, a major goal of the Uruguay Round trade negotiations conducted by the General Agreement on Trade and Tariffs (GATT) was the elimination of so-called nontariff barriers to trade. In the case of quantitative restrictions in sectors such as agriculture, reform is to take place in two separate phases. The first phase involves the replacement of quota policies with tariff policies that have similar effects. The second phase then works to reduce the tariffs. The justification for the two-stage trade reform has been that immediate removal of restrictions may cause great domestic

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disruption. The success of GATT in achieving tariff reduction argues for the initial change to a tariff-based policy.[1]

Throughout the 1980s, New Zealand employed auctions to distribute the rights to import products from abroad. The data from these quota-license auctions provide an opportunity to test the validity of the presumed advantages of auctions. Quota rights were auctioned yearly or half-yearly for ten years. The rights were tradable and, indeed, an active secondary market developed. The presence of the secondary markets raises issues that are the focus of this article and relate directly to three main functions of an auction mechanism. First, the simple fact that retrade occurred suggests that the auctions by themselves were not successful in generating an *ex post* efficient allocation. Second, the secondary markets also provide data that help to test the revenue-raising role of auctions. We illustrate that resale prices for the quota rights tend to be substantially higher than the auction prices. This fact suggests that some potential revenues were forgone by the government authorities and accrued instead to the participants in the auctions. Finally, the fact that the auction prices tended to be below the *ex post* market value of the quota also places into doubt the informational role of the auctions. For some environments, the auction prices provide tight estimates for the range of the tariff equivalent. However, these same conditions suggest that resale prices and auction prices should be similar. The fact that resale prices tend actually to be higher suggests either that the auctions understate the true tariff equivalent or that the resale prices are substantial overestimates of the equivalent.

In the next section we construct a simple model of auctions with resale. In Section 3 we show that quota auctions can serve an important informational role. The mean of the equilibrium prices determined at these auctions bounds the mean of the tariff equivalent for the product for which the quota license is offered. In Section 4 we describe more fully the New Zealand auctions and the data they generated. In Section 5 we compute the bounds implied by the theoretical analysis. We then use subsequent secondary market prices for these quota licenses as proxies for the true value of the license and show that the auction prices tend to fall below these values. This phenomenon both suggests a rejection of the theoretical predictions and indicates the presence of some surprisingly high arbitrage possibilities.

The auctions in the theoretical section are idealizations of the true institutions. In Section 6 we examine various alternative explanations for the deviation of auction prices and resale prices. An important departure from reality is our assumption, in the theory, that bidders desire to purchase fixed and identical quantities and may submit bids only in single units. In fact, as in many auctions, bidders were able to submit bid schedules and often purchased quite a variety of quantities. Little is currently known about equilibria of auctions of this form. However, it is known that in auctions in which bidders may have different marginal valuations for different quantities, incentives develop for bidders to reduce their stated demands (Ausubel and Cramton, 1998). The failure to predict accurately the relationship between auction prices and resale prices may be due to the fact that most bidders desire multiple units and are able to bid for them. However, we do not find strong evidence to indicate that multiple unit bidding is at fault. Neither is there strong evidence to indicate that the common-value component introduced by a secondary market is responsible. We provide a series of regressions that illustrates the correlation between the deviation of aftermarket prices and auction prices and various proxies for the aftermarket and multiple unit effects. We conclude tentatively that speculative purchases at auction followed by opportunistic

[1] For a description of the new trade agreement's tariffication provisions in agriculture, see Josling (1994).

pricing in the aftermarket led to higher than competitive aftermarket prices. Section 7 concludes. All proofs are in Appendix A.

2. The model

■ Fix a particular good for which quota rights are sold at auction. For a given auction, t, there are n^t bidders, each desiring a single unit of the q^t objects up for auction. We assume that the bidders may demand at most one unit primarily because little is known in auctions where bidders may choose the quantities they demand as well as the prices they submit. This assumption may be a significant one, and we examine further its role in Section 6.

For bidder i, the value of the right to import a unit of a good is given by

$$U_i^t = p_d^t - p_w^t - c_i^t,$$

where p_d^t and p_w^t are the realized domestic and world prices for the product in period t and c_i^t is the cost of bringing the product to the domestic market and distributing it. As is suggested by the subscripts, it is reasonable to expect that all firms in a given market care similarly about the world and domestic prices, while the cost of distributing the good enters only their utility.[2] For each auction a random vector, (z^t, x^t), that affects the valuation of the quota right is realized before the auction is held. The component z^t is publicly observed. It reflects information about the realization of world and domestic prices and may also contain auction-specific information, such as the number of licenses for sale and the number of bidders at the auction. The component $x^t = (x_1^t, \ldots, x_{n^t}^t)$ comprises the private information of each bidder, $i = 1, 2, \ldots n^t$, about his true costs. Throughout we assume that x_i^t is a single-dimensional real number. We also assume that all random variables are independent across periods and that bidders in any given auction play the one-shot Nash equilibrium. These assumptions remove informational or strategic links across auctions. Except where necessary for clarity, we drop the superscript t in what follows.

Given the assumption that each firm may acquire either one unit of quota licenses or none, then in a market where q of these quantities of quota licenses are sold, the value of the qth-highest valuation approximates the realized tariff equivalent for that market in that year. To see this, let $U_{(q)}$ denote the qth-order statistic of the U_i's. For later use, we define $X_{(j)}$ to be the jth-order statistic of the signals of the n bidders, and $Y_{(q)}$ is the corresponding order statistic of the signals of the $n - 1$ bidders excluding bidder i. If, instead of a quota, a tariff $\tau \in [U_{(q+1)}, U_{(q)}]$ had been imposed, then only the importers with values $U_{(q)}$ or higher would be able to import the good profitably in that year. In this sense, any such τ would mimic the effects of the quota regime. We ignore the relatively inconsequential multiplicity implied by the fact that $U_{(q+1)} < U_{(q)}$, since any selection from that interval would have the same economic effects. More problematic from a policy maker's perspective is the fact that from year to year this interval varies randomly. The tariff equivalent is a random variable. We focus on the information that auctioning of quota rights over time provides about the distribution of this random variable.

The randomness of the underlying market makes each bidder's private information of value to themselves, to other participants in the auction, and to a policy maker who may wish to use this information to glean a better idea of the tariff equivalent. The

[2] If bidders expect to import the good to different, isolated markets, p_d could also be bidder dependent. This feature does not change the analysis qualitatively, so we ignore it in the remainder of the article.

nature of the private information that the bidders enjoy will have consequences for their bidding behavior and for the information that the bids reveal. The possibility of retrade on a secondary market makes it important to be explicit about timing. We assume that bidders' signals, x_i, are learned before the auction takes place. The *true* values, U_i, are realized at some point after the auction. Since the initial information may yield only imprecise signals about the true valuations, it is possible that after the final resolution of uncertainty there remains an incentive to reallocate the licenses. It is this incentive that motivates the secondary markets.

We assume that once the true values are realized they become public knowledge and that only the bidders in the original auctions participate as end users in the secondary markets. The first assumption rules out the possibility of strategic behavior in auctions geared toward affecting subsequent prices on resale. The second assumption appears to be borne out by the information we have about the secondary markets.[3] If the resale prices do, in fact, represent the true value of the quota licenses, then the price will lie somewhere between $U_{(q)}$ and $U_{(q+1)}$ when there are q licenses available. Reflecting the assumption that the secondary markets are competitive, throughout the remainder of the article we assume that retrade occurs at a price that yields the $q + 1$st-highest valuer exactly zero surplus.[4]

This characterization of the behavior of the secondary markets is a simplification of the approach used by Haile (1996). He does not assume that the final values are public knowledge and allows for strategic behavior in the secondary market as well. This, in turn, generates an additional strategic effect in the auction stage that is reminiscent of the ratchet effect (Freixas, Guesnerie, and Tirole, 1985). In Haile's analysis, this effect raises bids. However, the direction of that effect appears to be a consequence of the way the secondary market game is modelled, and in general, higher or lower bids are possible. This indeterminacy, the large number of bidders, and the desire for tractability serve as the rationale for ignoring this "ratchet" effect here.

Let $dF_s(\cdot|r)$ denote the density of a random variable s conditional on a realization of event r. We make the following assumptions about the joint distributions of the various random variables:

Assumption 1. All bidders share the same information, z.

Assumption 2. Given z, for any $j \neq i$, and for any x_i and x_j, U_i and U_j are statistically independent. If $x_i = x_j$, U_i and U_j have the same conditional distribution.

Assumption 3. Given z, for any $j \neq i$, x_i and x_j are distributed independently and identically and have strictly positive densities over a compact, convex support, which we set to be [0, 1].

Assumption 4. For any i, given z, let the density of U_i conditional on x_i be $dF_{U_i}(\cdot|x_i)$, defined over the unit interval, [0, 1]. $dF_{U_i}(\cdot|x_i)$, is assumed to be strictly positive and to satisfy the monotone likelihood ratio condition

$$\forall u \leq v, \qquad x \leq y, \qquad \frac{dF_{U_i}(u|x)}{dF_{U_i}(u|y)} \geq \frac{dF_{U_i}(v|x)}{dF_{U_i}(v|y)}.$$

These assumptions place restrictions on the class of environments. The publicly

[3] An informal survey of the (handwritten) secondary market entries revealed that many of the transactions were with names that also appeared in the list of bidders.

[4] The analysis of the article would also follow if the resale price was a constant convex combination of $U_{(q)}$ and $U_{(q+1)}$.

observed variable, z, may be thought of as conditioning the distribution of the world price, p_w, and domestic price, p_d. These prices are assumed to be independent of the bidders' private information, x_i, for all i. Assumption 2 implies that *conditional on the public information about prices and other elements of z,* the final valuations for the licenses are independent and private. The active secondary market supports the claim that there is a substantial private-value component in these auctions, since a pure common-value environment would eliminate any incentive to redistribute the good among the buyers. At the same time, this market provides an endogenous source of commonality of valuations at the auction stage. The variation among bidder valuations is explained as deriving from bidder-specific costs. A similar assumption, Assumption 3, is made for the private signals of the bidders. In keeping with the interpretation of x_i as a signal of the final value of U_i through its information about costs, x_i and U_i are assumed to be jointly distributed (Assumption 4). The assumptions also imply that i's valuation, U_i, is independent of j's signal x_j.

Although the model is special, it is not unreasonably restrictive. The secondary market introduces a source of a common value in a very natural way. The fact that all bidders care equally about potentially random world and domestic prices does not on its own generate an affiliated-values model in the sense of Milgrom and Weber (1982) as long as bidders have similar access to whatever information there is about the realization of world and domestic prices. As long as bidders all share the same imperfect information about future prices, this source of uncertainty will merely shift all bids accordingly to take the information into account. Since this information plays no distinctive role in the analysis of strategic behavior, we ignore it in the theoretical analysis. Its relevance arises for the empirical analysis, since it explains variation across auctions of different goods and at different times.

3. Equilibrium bidding behavior in discriminatory auctions

■ In our theoretical version of New Zealand's discriminatory price auction, bidders submit bids for single units of the good simultaneously and the top q bidders obtain the object. Winners pay the price they bid and the payment is made immediately.

Fix a bidder, i, and define the function, $w(\cdot, \cdot)$ by

$$w(x, y) = E[U_{(q+1)} | X_i = x, Y_{(q)} = y].$$

The combination of the monotone likelihood condition and independence implies that $w(\cdot, \cdot)$ is increasing in both its arguments. The following result will prove useful.

Theorem 1. $E[w(X_{(q+1)}, X_{(q+1)})] \leq E[U_{(q+1)}] \leq E[w(X_{(q)}, X_{(q)})]$.

Let $B_q(x)$ satisfy

$$B_q(x) K_{Y_q}(x) = \int_0^x w(y, y) k_{y_q}(y)\, dy,$$

where $K_{Y_q}(\cdot)$ is the cumulative distribution function of the qth-highest of the signals of the $n - 1$ other bidders and k_{Y_q} is its associated density function. Assumptions 1–4 imply that $Y_{(q)}$ is independent of bidder i's signal. The next result is an adaptation of standard proofs (see, for example, Milgrom and Weber, 1982).

Theorem 2. $B_q(x)$ is a monotonic, symmetric equilibrium bidding function in the q-unit discriminatory auction.

Observe that the function $w(x, x)$ represents a symmetric equilibrium bidding function for a uniform $q + 1$st-price auction. The assumption that signals and valuations are independently distributed across bidders yields a useful result that is well known in other contexts.

Theorem 3 (revenue equivalence).

$$E[w(X_{(q+1)}, X_{(q+1)})] = \sum_{i=1}^{q} \frac{E[B_q(X_{(i)})]}{q}.$$

Revenue equivalence follows because the right side represents the expected average price paid in the discriminatory auction. Theorem 3 can now be used to derive bounds for the tariff equivalent in the New Zealand discriminatory auctions.

Theorem 4. The expected value of the average winning bids in the q-unit discriminatory auction is less than or equal to the expected value of $U_{(q+1)}$.

Theorem 4 provides a lower bound on the tariff equivalent in the case of discriminatory auctions. The lack of a simple upper bound stems from the common-value component that the secondary markets introduce. For some common-value auctions, it is possible that all higher bids lie below the tariff equivalent. However, the source of the common value is of a special kind in this environment, and an upper bound for the tariff equivalent is available if an additional assumption is made. The next result indicates that if the link between initial signals and final valuations is strong enough, then we can derive an upper bound in each auction. Holding the distribution of the U_i's fixed, consider a sequence of environments, $m = 1, 2, \ldots$, specifying a progressively stronger relationship between signals X_i and valuations U_i such that

$$\begin{aligned} \lim_{m\to\infty} \text{Prob}^m[U_{(q+1)} > U_i | X_i = x, Y_{(q)} \le x] = 0, \quad \forall x, \\ \lim_{m\to\infty} \text{Prob}^m[U_{(q)} < U_i | X_i = x, Y_{(q)} \ge x] = 0, \quad \forall x, \end{aligned} \tag{1}$$

where $\text{Prob}^m[A|B]$ is probability of event A given event B for environment m.

Theorem 5. For a sequence of environments $m = 1, 2, \ldots$ satisfying (1),

$$\lim_{m\to\infty} \frac{\sum_{i=1}^{q} E_m[B_q(X_i)]}{q} = E[U_{(q+1)}] < \lim_{m\to\infty} \frac{\sum_{i=1}^{q-1} E_m[B_q(X_i)]}{q-1} \le E[U_{(q)}].$$

The limiting hypothesis of the theorem considers environments in which the initial signal about the object's true value becomes more accurate. In the limit, there is no randomness following the auction and, so, no motivation for retrade. In our dataset, conditional on a secondary market transaction occurring, the average quantity transacted was only 6% of the total quantity sold. Since we record only secondary sales of one retrader, this figure is only a lower bound on the total sales, so we cannot say very much about the overall frequency of recourse to the secondary market. On the assumption that their overall use remained relatively small, Theorem 5 implies that the expectation of the average of the highest winning bids excluding the marginal winning bid exceeds the expectation of the tariff equivalent. We use this result to construct a testable conjecture concerning an upper bound on the tariff equivalent.

4. Auction data and institutions in New Zealand

■ New Zealand auctioned the rights to import restricted products from 1981 to 1991. Licenses could be traded, and indeed there was an active resale market in import quota licenses. The model indicates that the aftermarket has two important economic effects. First, it introduces the possibility of a common-value element into the environment even if none were present initially, for bidders now have the opportunity to purchase a license *ex post* at a common market price that is unknown at the time of the auction. Second, resale prices themselves provide a basis for estimating the value of the licenses, and the tariff equivalent.

The New Zealand government auctioned import quota rights for over 400 quite specific categories of goods, including such disparate items as underwater spear guns and meat offal. The auction system was initially intended to serve the purpose of information gathering, a so-called tariff-testing mechanism for the process of tariffication. Although the auction bids were not used to develop tariff equivalents, they often influenced the transition out of quotas, since categories with very low bids were typically liberalized first. New Zealand abolished all licensing arrangements by 1992. Auctions were held yearly or half-yearly. Sealed bids were invited for submission within two months of the announcement of the auction. The licenses were generally available for use soon after the auction. Any resident firm or person was eligible to participate.[5] Only prospective bidders who had previously defaulted were technically barred from participation in the auction, but even this limitation was not actually enforced. For most of the licenses, a unit of the quota represented the right to import $NZ 2,000 of a given good. Until 1988, licenses were valid for one year. After 1988, the expiration date on licenses was eliminated. Bids were submitted in nominal dollar amounts. Thus, the conversion to percentages was easily accomplished by dividing the bid by the value of imports allowed by the license unit. The auction was discriminatory, and the discriminatory auction described above fits the institution well, subject to the caveat that bidders could bid on more than one unit. Bids were ordered from highest to lowest. In this way a form of market-demand curve was constructed, and the intersection of this curve with the quantity of licenses available determined the winners of the licenses. Winners were expected to pay for the license immediately following the auction.[6] Results of the auction were published in the *New Zealand Gazette* soon after the auction. This publication is the source of our auction data from New Zealand. Because of the amount of work involved, we transcribed only a subset of categories of goods from the *Gazette.* The data included the date of the auction, the total amount tendered, all bids submitted, and quantity asked for, as well as the total number of bidders.

Table 1 provides some summary statistics of this dataset. The number of auctions and products auctioned should not necessarily be taken as informative of New Zealand's overall policy. It simply indicates what was recorded in the dataset at our disposal. However, as can be seen in the average quantities tendered per auction, New Zealand increased the number of licenses offered in the auctions over the last half of the 1980s.

[5] ''Person'' was interpreted rather liberally in New Zealand. At one point, a bidder submitted a bid under the name of his dog and showed up with the pet when the animal won.

[6] The government responded in 1986 to a substantial number of defaults by imposing security deposits. These succeeded in ending the defaults. Therefore, we report only the data from the years 1987 to 1990. Data from 1991 were also not reported, since the program was halted in that year.

TABLE 1 Summary Statistics for New Zealand Import License Auctions

	Number of Auctions	Number of Products	Units Tendered	Bids per Auction	Number of Winning Bids	Clearing Percentage
1987	80	48				
Average			597.6	173.6	24.1	13.3
Maximum			4,656.0	1,196.0	299.0	71.0
Minimum			13.0	3.0	1.0	0
1988	82	50				
Average			803.4	85.3	33.2	9.9
Maximum			5,429.0	573.0	180.0	71.2
Minimum			17.0	15.0	2.0	0
1989	33	28				
Average			1,103.4	91.4	44.5	18.4
Maximum			6,898.0	408.0	249.0	76.0
Minimum			18.0	7.0	1.0	0
1990	24	23				
Average			1,488.1	107.0	60.1	12.6
Maximum			8,950.0	361.0	173.0	55.3
Minimum			22.0	29.0	3.0	.1

Nevertheless, the lowest price paid (in percentage terms) remained fairly high, as did the average price paid. This probably reflects the practice of the New Zealand government to eliminate quota restrictions on the products that attracted only low bids in previous auctions. In some years more than one auction per product was held. These are treated as independent auctions. Thus, averages, maximums, and minimums are computed across all the auctions in the dataset for a given year. (Auctions where there was excess supply—fewer than the total quantity offered were demanded—were dropped from the dataset.) The ‘‘number of winning bids’’ column reports the number of distinct bids submitted. Since many bidders submitted a number of bids at different prices, these values generally exceed the number of bidders. The data are recorded by bid rather than bidder, and the size of the dataset made it infeasible to determine the number of different bidders per auction for all auctions. However, this number was computed for a subset of the auctions and is reported later in Tables 3 and 4. There it can be seen that each bidder submits, on average, about two bids per auction. The ‘‘clearing percentage’’ column in Table 1 reports summary data on the lowest winning bid per auction, calculated as a percentage of the value of imports.

Theorem 4 shows that the expected value of an average of all winning bids lies below the expected value of the tariff equivalent. Theorem 5 implies that if the link between initial signals and final valuations is strong enough, the expected value of an average of all winning bids excluding the lowest accepted bid lies above it. We adapt these results to allow for multiunit bids by computing weighted averages of bids, using as the weights the quantity requested per bid. Some bidders submitted multiple bids. By averaging across all winning bids in this way, we effectively treat each distinct bid as coming from a different bidder. We return to this issue in Section 6. Table 2 presents simple averages across auctions in each year of the weighted average of all winning

TABLE 2 Predicted Upper and Lower Bounds of Tariff Equivalent

Year	$\frac{1}{N^t} \Sigma_k b_k^t$	$\frac{1}{N^t} \Sigma_k B_k^t$	Difference	Standard Deviation
1987	16.81	17.55	.74	.87
1988	14.08	15.79	1.72	1.88
1989	25.57	26.90	1.33	1.44
1990	20.27	21.63	1.36	1.79

Note: Column 2 is the yearly average across all auctions of the quantity weighted average winning bids for the given year. Column 3 is the yearly average of the quantity weighted average of winning bids excluding the lowest winning bid. The last two columns are the average difference and the standard deviation of the difference. (N^t is the number of auctions recorded in our dataset in year t.)

bids[7] and the weighted average of all winning bids excluding the lowest winning bid (B_k^t). Since B is not defined in the case of a single winning bid, these auctions are not included. Over the entire dataset, the simple average of b_k^t represented an average tariff of 17.5% while the simple average of B_k^t was 18.8%, yielding an average difference of 1.3 percentage points with a standard deviation of 1.6. These summary data indicate that under the hypothesis of Theorem 5, data from these auctions can provide very tight bounds for estimating an appropriate tariff equivalent. In the next section, though, additional data from secondary market transactions provide some reason to doubt the conditions underlying these results.

5. Secondary market data in New Zealand

■ There was no formal market for transfers of auctioned import licenses in New Zealand. Auctioned import licenses could be transferred, but until late 1987, import license transfers had to be processed through a government agency. No records of prices and quantities for trades on the secondary market were compiled by government agencies or industry organizations. An informal market, which expanded after 1987, existed among retailers, importers, customs agents, and quota brokers. Our source of data for secondary market trades in New Zealand is the written records of Mertz and Associates, one of the major quota brokers in Auckland. Mertz and Associates, a customs agency primarily serving apparel importers, frequently had to transact in auctioned import licenses on behalf of clients, and it also participated in the auctions and secondary market on its own account. The dataset consists of 1,447 secondary market transactions from October 1987 through September 1991. It includes information on the date of the original auction as well as the date of the secondary market transaction, commodity category, buyer, seller, quantity bought or sold, and price. Despite a search, we could not find any other major quota broker who had preserved records of transactions. Although some products had more than one auction per year, since the data recorded the original auction, we could compute directly the days that elapsed between the auction and the resale. In our dataset of secondary market transactions, there were four products with multiple auctions in 1989 and one with multiple auctions in 1990. In the subsequent regression analysis these observations are not an issue, since we restricted attention to product categories for which resale data was available in all four years and none of these products satisfied that criterion.

[7] That is, the simple average of b_k^t, which is the weighted average of all winning bids in auction k in year t.

Table 3 provides some summary statistics of the secondary market data. The number of secondary market transactions generally rose over the period, as did the proportion of total available quantity that was transacted. (This is only an imperfect measure of total transactions, since there were other trades that Mertz was not involved in.) The average number of days after the auction that the secondary market transaction occurred was fairly long, around 200 days. This figure remained fairly constant over the period. The number of products for which we have data on the aftermarket sales tracks roughly the data we have on the products that were made available at auction. The column ''secondary market sales'' reports the total number of transactions in a given year across auctions for that year and the average, maximum, and minimum number of transactions across auctions. To construct the percentage of total sales, we first computed the total amount of quota resold in the secondary market for a given auction and divided it by the total amount sold at the original auction. The average, maximum, and minimum across auctions for a given year is then reported. The ''days after auction'' column computes the days elapsed between the specific quota auction and each recorded resale and reports averages, maximum, and minimum across all secondary market transactions. The final three columns report auction-specific data. ''Number of bidders per auction'' is the number of different bidder names. Since bidders may have entered bids under different names, this variable may overstate the number of bidders. The ''number of bids per bidder'' column is the total number of distinct bids divided by the number of bidder names. Our assumption that each bidder submits only a single bid is not borne out in practice. Similarly, the column reporting Gini coefficients of quantities demanded per bid indicates that bidders often bid for varying quantities of licenses.[8]

In Table 4, attention is restricted to the 10 product categories for which we have secondary market transaction data in every year. Mertz and Associates dealt mainly in textiles. The product categories we have reflect this. There is some variability across product codes in the frequency at which the secondary market was employed, the average date to transaction, and especially in the proportion of the quota sold at auction that was ultimately sold in the aftermarket. By far the highest aftermarket sales as a percentage of total sales occurred in category 916, women's and girls' windjackets. Since Mertz dealt mainly in clothing, the high quantity resold for category 916 likely reflects this bias.

If secondary market prices reflect more accurate (though random) measures of the true quota value, Theorem 4 and Theorem 5 also provide a natural test of the discriminatory-price auction model. Does the average secondary market price lie between b and B? In order to implement a test of this prediction with the data, we make the following specifications. Let π^t_{kj} denote the sales price in the jth secondary market transaction for good k in auction t discounted to the time of the auction at a 10.5% annual interest rate (compounded daily).[9] Let b^t_k and B^t_k denote the weighted average of the winning bids and the winning bids excluding the lowest winning bids, respectively, for the tth auction of the kth good.[10] The differences in the indices of π^t_{kj} and the bid averages indicate a slight complication. Since there are typically many secondary sales per auction, we have more observations of secondary market prices than of

[8] A Gini coefficient of (almost) zero corresponds to an auction where every bid submitted demanded the same quantity. The samplewide average Gini coefficient was .657.

[9] A daily interest rate of 10/365 yields this annual rate. The average rate of change of wholesale prices from 1987 to 1991 in New Zealand was 5.1% (International Monetary Fund, 1993). We also used a 20% discount rate, with similar results.

[10] In the restricted dataset, each good is auctioned only once in each year, so each index pair k, t uniquely defines an auction of a good k in year t.

TABLE 3 Auctions and Secondary Markets

Year	Number of Products	Secondary Market Sales	Percentage of Total Sales	Days After Auction	Gini of Bid Quantities per Auction	Number of Bids per Bidder	Number of Bidders per Auction
1987 (total)	30	191					
Average		6.4	6.5	200.6	.7	2.2	110.0
Maximum		17.0	28.4	318.0	.9	4.6	366.0
Minimum			0	15.0	.1	1.2	14.0
1988 (total)	33	270					
Average		8.2	4.6	191.5	.7	1.8	56.8
Maximum		35.0	15.2	451.0	.9	3.3	101.0
Minimum			.1	47.0	.5	1.1	24.0
1989 (total)	27	368					
Average		13.9	10.5	184.1	.7	1.4	77.0
Maximum		64.0	40.3	385.0	.8	3.7	383.0
Minimum			0	3.0	.5	1.0	11.0
1990 (total)	20	576					
Average		28.8	14.8	210.0	.8	2.4	44.0
Maximum		96.0	111.3	408.0	.9	3.5	112.0
Minimum			.4	0	.5	1.8	17.0

auction prices. A natural way to resolve this is to create additional b's and B's as follows. For all k, t, and j, let $b^t_{kj} = b^t_k$ and $B^t_{kj} = B^t_k$. The stochastic process generating the random variables in our dataset is assumed to be given by

$$b^t_{kj} = \overline{b} + \epsilon^t_k + \eta^t_{kj},$$

$$B^t_{kj} = \overline{b} + \epsilon^t_k + \eta^t_{kj} + \nu^t_{kj}, \qquad \nu^t_{kj} \geq 0,$$

$$\pi^t_{kj} = \overline{\pi} + \epsilon^t_k + \kappa^t_{kj}.$$

This specification is very general. We make no assumptions about the distribution of ϵ^t_k. Its distribution may depend on k and t, so we allow for a year- and good-specific term that affects both the auction prices and secondary market prices similarly. This term could include changing expectations about the world or domestic price. Its mean need not be zero. The random variable ν^t_{kj} is an auction-specific component that reflects the gap between b^t_k and B^t_k. Since $B > b$, it must be positive. The variable η^t_{kj} is also an auction-specific random variable. By construction, we must have $\eta^t_{kj} = \eta^t_{kj'}$ for all j, j', k, and t. The components ϵ, η, and ν capture year-, product-, and auction-specific variables contained in the commonly observed variable, z^t_k.[11] The variable κ introduces secondary market price variation. Thus, bids and prices may vary with the product or the year of the auction, and we put no restrictions on the distributions except, where

[11] That is, observed by the bidders though not by us as researchers or policy makers.

necessary, some limited independence and identity of the distribution of the error terms across either goods or periods and across secondary market transactions.

Subtract b^t_{kj} from both the secondary market transactions and B^t_{kj} to form the series $\pi^t_{kj} - b^t_{kj}$ and $B^t_{kj} - b^t_{kj}$ respectively. Theorem 4 and Theorem 5 imply that $0 \leq E[\pi^t_{kj} - b^t_{kj}] \leq E[B^t_{kj} - b^t_{kj}]$. Given a random sample of observations of bids and prices, the sample means provide estimates of the true means. We use this observation to construct our test statistic.

The model implies that the parameter of interest, $E[\pi^t_{kj} - b^t_{kj}]$, lies in a nontrivial real interval. However, for a two-sided test of the hypothesis that a parameter lies in the interval, $[0, \Delta_i]$, the upper bound on the confidence interval for this test is lower than the upper bound for a two-sided test that the mean of $\pi^t_{kj} - b^t_{kj}$ *equals* the mean of $B^t_{kj} - b^t_{kj}$. Given the high frequency of rejections from this less stringent test, we conduct only the latter test. Thus a test of the hypothesis implied by Theorem 5 can be conducted by computing the sample mean of $\pi^t_{kj} - b^t_{kj} - B^t_{kj} + b^t_{kj} \equiv \pi^t_{kj} - B^t_{kj}$ and testing whether or not it is significantly different from zero. Since the model implies that this statistic should be negative, finding it to be significantly positive must imply a rejection of the model's prediction. Notice that the ϵ^t_i's disappear. This is why we can specify its distribution arbitrarily. We can use our data with minimal assumptions on the distributions of the random variables by disaggregating either by year or by product as follows. Holding t fixed, we suppose that η^t_{kj}, ν^t_{kj} are independently and identically distributed across products, k, and that κ^t_{kj} are independently and identically distributed across products k and secondary market sales, j.[12] We have an unbalanced ''panel'' dataset where the π^t_{kj} observations can be grouped by product categories for each regression. By estimating four random-effects equations,[13]

$$\pi^t_{kj} - B^t_{kj} = CONSTANT^t - (\nu^t_k + \eta^t_k) + \kappa^t_{kj}, \qquad t = 87, 88, 89, 90,$$

we test whether the constant is significantly different from zero for each equation. The results from this regression are printed in Table 5. In case the data-generation process we have assumed in the secondary market actually overstates the number of observations at our disposal (for example, if $\mathrm{Var}[\kappa^t_{kj}] = 0$), we also report the between regressions using the mean secondary market prices. The true implication, of course, is that the constant should be nonpositive; however, in all years the estimate proves to be significantly positive and so, *a fortiori*, the implication of Theorem 5 is rejected.

An alternative disaggregation can be done by product code. That is, hold k fixed and run a regression similar to the one above for each product code. There are ten products for which secondary market transactions are available in all four years. The regression results are reported in Table 6. With the exception of product codes 916, 924, and 935, the implication of Theorem 5 is rejected. For product 924, we cannot reject the null that the constant is zero. For 916, though, the random-effects estimate is significantly negative. In fact, by reversing and testing the stronger implication of Theorem 4 that $\pi^t_{kj} - b^t_{kj}$ is positive, as is done in the line for 916A, we also find a rejection. There is no observable distinction for this product category except the fact that an unusually high proportion of products sold at auction were retraded. This is puzzling, since the assumption underlying Theorem 5 suggests that the lower bound will be more easily satisfied the less stable the initial allocation from the auction.

[12] This can be generalized. It should be evident that if we defined $\kappa^t_{kj} \equiv K^t_k + L^t_{kj}$, we could allow for some heteroskedasticity across the products for κ as well.

[13] For a description of how these models are estimated, see Greene (1990).

TABLE 4 Secondary Markets and Auctions by Code

Year	Secondary Market Sales	Percentage of Total Sales	Days After Auction	Gini of Bid Quantities per Auction	Number of Bids per Bidder	Number of Bidders per Auction
902 (total)	55					
Average	11.0	2.8	235.7	.6	2.0	65.8
Maximum	19.0	6.0	406.0	.9	3.6	90.0
Minimum	2.0	.2	0	.1	1.2	45.0
912 (total)	85					
Average	17.0	10.0	209.0	.6	2.0	57.2
Maximum	31.0	20.9	406.0	.8	3.0	71.0
Minimum	4.0	.7	0	.1	1.0	48.0
913 (total)	106.0					
Average	21.2	6.8	199.2	.6	2.2	100.4
Maximum	56.0	12.8	406.0	.9	3.5	154.0
Minimum	6.0	.7	0	.1	1.1	56.0
916 (total)	51.0					
Average	10.2	33.3	190.0	.5	1.9	53.0
Maximum	37.0	111.3	273.0	.7	2.7	79.0
Minimum	1.0	1.2	2.0	.1	1.3	35.0
921 (total)	79.0					
Average	15.8	4.5	174.7	.7	1.6	119.8
Maximum	33.0	12.9	406.0	.9	2.4	340.0
Minimum	1.0	.2	8.0	.2	1.1	31.0
922 (total)	192.0					
Average	38.4	9.0	195.3	.7	1.8	154.8
Maximum	96.0	20.6	406.0	.9	3.0	366.0
Minimum	4.0	.7	0	.1	1.2	66.0
924 (total)	131					
Average	26.2	13.3	216.8	.6	1.7	106.4
Maximum	60.0	27.4	406.0	.8	2.5	246.0
Minimum	1.0	.4	2.0	.1	1.2	47.0
927 (total)	121					
Average	24.0	6.7	195.3	.7	2.1	151.2
Maximum	55.0	15.1	408.0	.9	3.7	356.0
Minimum	9.0	1.5	0	.1	1.2	72.0
935 (total)	59					
Average	11.8	12.5	170.8	.5	1.6	30.0
Maximum	26.0	36.0	406.0	.7	1.9	42.0
Minimum	2.0	.9	2.0	.2	1.3	19.0

TABLE 4 ***Continued***

Year	Secondary Market Sales	Percentage of Total Sales	Days After Auction	Gini of Bid Quantities per Auction	Number of Bids per Bidder	Number of Bidders per Auction
940 (total)	33					
Average	6.6	11.4	201.2	.6	1.7	48.6
Maximum	17.0	32.4	385.0	.8	2.8	85.0
Minimum	1.0	.1	1.0	.1	1.1	21.0

Code List

Code	Description
902	Men and boys jerseys
912	Men and boys outergarments, other
913	Women and girls jerseys
916	Women and girls windjackets
921	Women and girls skirts
922	Women and girls shirts
924	Women and girls outergarment
927	Men and boys shirts
935	Women and girls vests
940	Shawls and scarves

The whole dataset can be utilized by assuming that all the random variables are independently and identically distributed across k and t. The value of the mean difference is 5.7 with a t-statistic of 17.4, implying an easy rejection of the null. Over the whole sample of auctions for which secondary market transactions are available, the sample mean of B_k^t is 26.0%, while the sample mean of π is 33%. This represents a

TABLE 5 **Secondary Market Prices Compared to Auctions Prices by Year**

Year	Total Number of Observations	Maximum Observations per Product (k)	Minimum Observations per Product (k)	Between Estimate	Random-Effects Estimate
1987	191	17	1	10.5 (4.7)	10.4 (5.0)
1988	270	35	1	4.2 (2.2)	3.4 (3.0)
1989	368	64	1	6.1 (2.3)	5.8 (3.5)
1990	576	96	1	5.7 (2.0)	6.1 (2.9)

t-statistics in parentheses.

TABLE 6 Secondary Market Prices Compared to Auctions Prices by Product Code

Code	Total Number of Observations	Maximum Observations per Year	Minimum Observations per Year	Between Estimate	Random-Effects Estimate
902	53	19	7	10.8 (3.1)	10.6 (4.4)
912	81	31	8	7.3 (2.4)	7.3 (2.4)
913	100	56	8	9.5 (4.0)	8.8 (4.3)
916	50	37	2	−8.6 (−1.2)	−9.5 (−2.8)
916A	50	37	2	−6.9 (−1.0)	−7.8 (−2.4)
921	78	33	11	16.3 (2.6)	16.3 (3.7)
922	188	96	11	7.6 (2.6)	8.3 (3.6)
924	130	60	17	4.1 (1.0)	4.0 (1.2)
927	109	55	9	6.8 (2.6)	7.3 (3.0)
935	59	26	2	3.0 (.6)	4.4 (1.8)
940	32	17	2	11.4 (5.9)	9.5 (5.7)

Note: *t*-statistics in parentheses. 916A reports the regression $\pi^t_{916j} - b^t_{916j} = CONSTANT^t$.

secondary market premium more than 26% higher than the weighted bids in the auction, suggesting some very tempting arbitrage possibilities!

6. Discussion

■ The New Zealand data reject the implication of Theorem 5. What accounts for the deviation between the auction price and the later resale price? A variety of joint hypotheses underlie the theoretical model. In this section we discuss which of the maintained hypotheses may fail to hold.

One explanation for the failure of the model to predict accurately the relationship between bids and resale prices is the fact that bidders were able to, and did, submit multiple bids. Does the presence of a resale market generate incentives to purchase excess units with the intention of reselling them on the market?[14] Consider a simple extension of the model in which bidders continue to have use for a single unit but may bid for as many as f units that they can resell on the secondary market. Suppose that $q/f = s$ is a whole number, so that winning bidders remain symmetric. Appendix B illustrates that in this simple extension, it remains the case that bidders with highest s signals win quotas and employ bids that satisfy the same lower bound as that given in Theorem 4.

[14] We are grateful to a referee for suggesting this possibility.

The opportunity to purchase speculatively in the auction and then resell in the secondary market changes the environment from an essentially private-value auction model to a common-value model. In an earlier version of this article, we show that the upper bound implied by Theorem 5 will no longer be valid if the auction is characterized by common values. Since winner's-curse effects can suppress bids in auctions, if there is a common-value element to the auction due to speculative purchases, this element may explain lower-than-expected bids. As information is revealed over time, the average market value of an object will typically rise as the winner's-curse effect is ameliorated. Both the auction itself and the passage of time between the auction and the secondary market sale could explain a further increase in such information. Nevertheless, if this is the explanation for the difference, it is interesting that bid suppression due to common-value effects accounts for a secondary market premium as high as 26%. In a simple, independent-signal, pure-common-value model where the value of each unit is the sum of the signals of the bidders and the number of units sold is approximately one-half the number of bidders, n, the ratio of the true expected value to the expected auction price in a qth-price auction is $(2n)/(2n - 1)$. This implies a premium of more than 20% only if the expected number of bidders is 3 or fewer.

The extension to allow speculative purchases, as well as the original model, restrict bidders to submitting constant per-unit bids and to bidding for similar quantities. The result is a strong symmetry in the model. Without symmetry, there are theoretical reasons to suspect that the ability to submit multiple-unit bids may be responsible for low auction prices. Ausubel and Cramton (1998) show that for many multiple-object auctions, whenever there are bidders with elastic demands for varying quantities, then equilibrium behavior must involve what they call ''demand reduction.'' Bidders submit something other than their true demand curves for the goods to induce a lower overall payment. An informal measure of demand-reduction behavior can be conducted simply by seeing how often marginal bidders also submitted higher bids. Over the years 1988 to 1990, bidders who won at the marginal bid also submit higher bids 27% of the time, implying that they were able to acquire the identical object at different prices.[15]

The possibility of demand reduction is important because the phenomenon is not simply a consequence of our inability to model auctions in which bidders submit demand schedules. The same effect is equally possible in institutions where many single-object auctions are held but which attract bidders with differing demand curves. The only difference in those cases is that the multiple-unit demand of the bidders is hidden by their participation in the many separate auctions held. Thus, the OCS oil auctions examined by Hendricks, Porter, and Boudreau (1987) and the eggplant auctions examined by Laffont, Ossard, and Vuong (1995) may be vulnerable to exactly the same demand-reduction behavior, even though bidders can submit bids in a given auction only for single units. It is, therefore, worth investigating the role of multiple-unit bidding more deeply.

In New Zealand, the actual auction institution deviated from our theoretical model in two conceptually distinct ways. First, bidders did not have to submit bids for identical quantities. Second, bidders could also submit multiple bids at different prices. Ausubel and Cramton (1998) suggest that either deviation could matter. We examined the importance of deviations from identical quantity bids indirectly by constructing two different measures of how much a given auction violated the identical unit demand

[15] A referee observed that if demand reduction is occurring, bidders who submitted some high bids in the auction, together with other low bids, might seek to acquire additional units through the secondary market. Unfortunately, our dataset does not allow us to check whether or not this is taking place, since we cannot compare secondary market trades with bidders in the original auction.

assumption: (i) a simple variance of quantities demanded across bids in an auction, and (ii) a Gini coefficient from the list of bids submitted. These were computed for auctions for which secondary market data were available. Summary statistics for the bid Ginis are reported in Tables 3 and 4. The second way the actual institution deviated from the theoretical game was that bidders were allowed to submit many bids. A simple measure of the single-bid assumption was constructed by dividing the total number of bids in a given auction by the total number of different bidders. These variables' summary statistics are given in Tables 3 and 4.

The upper bound derived from Theorem 5 requires the assumption that initial signals be informative enough about the final value to make resale relatively rare. Since we do not have information on all secondary market sales, we do not know how rare these transactions were. However, as Tables 3 and 4 suggest, Mertz, a major reseller, still sold a relatively small proportion of total quotas that were auctioned. Furthermore, Theorem 5 suggests that the less informative the initial signal, the less likely the upper bound will be to hold. We constructed a proxy to measure the informativeness of the signal, which is the proportion of the total quantity sold at auction that was later sold (by Mertz) on the aftermarket (the variable ''ratio''). Assuming that Mertz's frequency of resale is correlated with the total number of resales overall, we can presume that the higher this figure is, the less accurate (*ex post*) were the initial signals that determined bids. We regressed the difference between the average discounted secondary market price and B on ''ratio.'' We also included the number of secondary market transactions to see if there was some unexplained selection bias that creates a systematic relationship between the frequency of aftermarket sales and the aftermarket premium. The results of this regression, including some ''demand reduction'' variables, are reported in Table 7. In virtually all specifications, the estimated coefficient of ''ratio'' is significant but negative. This seems to argue against the conjecture that a failure of (1) is to blame. The more frequent the secondary market transactions, the more important the common-value component that is introduced by the aftermarket. This should make the auction environment less likely to be close to the limit in (1) and make the difference between secondary market prices and bids larger, not smaller. The reversal of the expected direction of this effect may indicate that the secondary market is, in fact, quite noncompetitive. The higher the relative demand on the aftermarket, the higher the price commanded by brokers in this market. No other variables were consistently significant, although the ratio of bidders to bids, a bid-reduction variable, was significant in some specifications.[16]

Could other factors be at play as well? While we cannot rule them out definitively, some informal arguments can be made against other explanations. An important simplifying assumption in the theory was our modification of Haile's (1995, 1996) model describing the secondary market transactions. However, Haile shows that when asymmetric information affects the secondary market trades, bids rise at the auction phase. If this effect is indeed present, we expect it to make it easier to find secondary market prices below the auction prices.

Could transactions costs in the secondary markets introduce selection effects that may bias the trading prices we observe? The effect of transactions costs on the data will depend on whether the price actually observed is inclusive or exclusive of these costs. Transactions costs operate like a tax. They lower the secondary market sell price and increase the secondary market buy price. The first effect is a selection effect in that only if the value of the potential seller is sufficiently low relative to the potential

[16] We also ran the regression excluding the apparently anomalous category 916, but there were no significant differences.

TABLE 7 **Regressions of Price Differences on Variables Representing Deviations from Model Assumptions**

Dependent Variable	Independent Variable: Ratio	Bidders/Bids	Number of Bidders	Variance of Bids	Number of Sec Sales	Number of Bids
PERDIF (R^2 = .12)	−4.7	−2.2	0	0	0	0
	(−2.5)	(−1.3)	(.6)	(−0.5)	(−.6)	(−1.6)
DIF (R^2 = .12)	−25.3	−17.0	.1	0	0	0
	(−2.7)	(−2.0)	(1.6)	(−1.6)	(0)	(−1.1)

DIF is average discounted secondary market price minus B^t_k. *PERDIF* is *DIF* divided by B^t_k. *t*-statistics are in parentheses. Sample size = 108.

buyer will a sale take place. If the price we observe is exclusive of these transactions costs, then the expected secondary market sale price will be *lower* than predicted by Theorems 4 and 5 and will, therefore, reinforce the conclusions drawn from the rejection of the hypothesis. If the price we observe includes all transactions costs, then the observed price will be higher than predicted, but because of the selection effect it will not rise by as much as the transactions costs. This latter effect is analogous to the well-known result that in a market with an upward-sloping supply curve, the purchase price rises less than one for one in response to the imposition of a tax. We do not have enough information to determine whether the price is inclusive or exclusive of transactions costs or, equivalently, whether we are observing a buy price or a sell price. The application of discounting already attempts to account for an obvious source of a transactions cost that is borne by the seller. Aside from that cost, though, we do not find strong arguments one way or the other that would allow us to conclude that this effect is responsible for the significant difference between the secondary market prices and the auction prices.

A further potential complication was the fact that the New Zealand government used low prices at auction as a criterion for removing quota restrictions. This introduces an additional, strategic complication, since domestic producers would have an incentive to bid up auction prices to forestall trade liberalization. Again, if present, this effect should be to bias bids in the auctions upward. Since our data suggest that bids are lower than expected, this effect only reinforces our conclusions.

The simplest explanation is that the secondary prices are not true tariff equivalents. A combination of speculative purchases at the auction by brokers and subsequent monopolistic pricing in the event of high aftermarket demand may have biased the secondary market prices upward. The negative effect of the ratio variable on the difference between secondary market prices and auction prices may support this interpretation. But although this provides an immediate explanation for the failure of auction prices to track secondary market prices, puzzles remain. What prevented the price differences from being arbitraged away? Entry into the quota auctions was quite unrestricted. The auctions appeared to provide a much lower-cost alternative to purchasing on the secondary market and incurring this high-cost alternative.

7. Conclusions

■ For a policy maker who is concerned that tariffication not raise the level of protection, the results of this article suggest that auction data may offer a useful lower bound on the tariff equivalent of a quota. We provide theoretical lower bounds that are supported empirically. The empirical and theoretical results also suggest that reliable upper bounds may be difficult to obtain. There are theoretical reasons to believe that more complicated multiple-unit preferences that bidders exhibit for quota licenses may bias bids downward, and the presence of active aftermarkets for the products may do so as well. However, the data from the New Zealand auctions suggest that skepticism is warranted about the informational value of aftermarket prices as well.

Appendix A

■ Proofs of Theorems 1–5 follow.

Proof of Theorem 1. By definition of $w(\cdot, \cdot)$ and the assumption that densities are nonatomistic,

$$\begin{aligned} w(x, x) &= E[U_{(q+1)} | X_i = x, Y_{(q)} = x] \\ &= E[U_{(q+1)} | X_i = x, Y_{(q)} = x, Y_{(q-1)} \geq x]. \end{aligned}$$

Furthermore, by affiliation,

$$w(x, x) \geq E[U_{(q+1)} | X_i = x, Y_{(q)} \leq x, Y_{(q-1)} \geq x].$$

Finally,

$$\begin{aligned} E[w(x, x) | x = X_{(q)}] &\geq E[E[U_{(q+1)} | X_i = x, Y_{(q)} \leq x, Y_{(q-1)} \geq x] | x = X_{(q)}] \\ &= E[U_{(q+1)}]. \end{aligned}$$

The equality stems from the fact that we are conditioning on the true state. Similarly,

$$E[w(x, x) | x = X_{(q+1)}] \leq E[E[U_{(q+1)} | X_i = x, Y_{(q+1)} \leq x, Y_{(q)} \geq x] | x = X_{(q+1)}].$$

Q.E.D.

Proof of Theorem 2. Fix a bidder, i. Let $V_{(j)}$ denote the jth-order statistic of the $(n - 1)$ U's excluding bidder i. Note that $Y_{(q)}$ is generally jointly distributed with $V_{(j+1)}$ and $V_{(j)}$ but by assumption is independent of X_i. Suppose that $B_q(\cdot)$ is a strictly monotonic, symmetric equilibrium bidding strategy. The expected utility of bidder i with signal x who submits, instead, a bid that would have been submitted by a bidder with signal $\tilde{x}$ is

$$\begin{aligned} &\int_0^{\tilde{x}} \int_0^1 [u_i - B_q(\tilde{x})] \, dF_{U_i}(u_i | X_i = x) k_{Y_{(q)}}(y) \, dy \\ &+ \int_0^{\tilde{x}} \int_0^1 \int_{u_i}^1 [v_{(q+1)} - u_i] \, dF_{V_{(q+1)}}(v_{(q+1)} | Y_{(q)} = y) \, dF_{U_i}(u_i | X_i = x) k_{Y_{(q)}}(y) \, dy \\ &+ \int_{\tilde{x}}^1 \int_0^1 \int_0^{u_i} [u_i - v_{(q)}] \, dF_{V_{(q)}}(v_{(q)} | Y_{(q)} = y) \, dF_{U_i}(u_i | X_i = x) k_{Y_{(q)}}(y) \, dy. \end{aligned}$$

To understand this expression observe that, by monotonicity, i wins whenever the qth-highest of the other signals is below $\tilde{x}$ and, conditional on winning at the auction, bidder i assures himself of the difference between his true value and the price to be paid for importing the good. In addition, whenever the $q + 1$st-highest valuation of the other bidders is above his true valuation, he will trade and capture the surplus represented by the difference between his valuation and $U_{(q+1)}$. If he fails to win at the auction, the possibility still remains that his final valuation is among the q highest of the bidders. By assumption, he will obtain a license and receive the surplus reflected by the difference in valuation between his value and the qth highest.

Differentiating this expression with respect to $\tilde{x}$ and imposing the condition that the resulting expression be zero at $x = \tilde{x}$ yields, as a necessary condition of any equilibrium,

$$B_q(x)k_{Y_{(q)}}(x) + \frac{dB_q(x)}{dx}K_{Y_{(q)}}(x)$$

$$= k_{Y_{(q)}}(x)\Big\{\int_0^1\int_{u_i}^1 v_{(q+1)}\,dF_{V_{(q+1)}}(v_{(q+1)}|Y_{(q)}=x)\,dF_{U_i}(u_i|X_i=x)$$

$$+\int_0^1\int_0^{u_i} v_{(q)}\,dF_{V_{(q)}}(v_{(q)}|Y_{(q)}=x)\,dF_{U_i}(u_i|X_i=x) \quad \text{(A1)}$$

$$+\int_0^1\int_{u_i}^1\int_0^{u_i} u_i\,dF_{V_{(q+1)}}(v_{(q+1)}|Y_{(q)}=x,\,V_{(q)}=v_{(q)})\,dF_{V_{(q)}}(v_{(q)}|Y_{(q)}=x)\,dF_{U_i}(u_i|X_i=x)\Big\}$$

$$= k_{Y_{(q)}}(x)w(x, x).$$

To derive the equality, note that the first term is the expected value of $V_{(q+1)}$ when it lies above U_i times the probability that this event occurs, the next is the expected value of $V_{(q)}$ when it lies below U_i times the probability that this event occurs, and the last is the expected value of U_i when it is the $q + 1$st-highest valuation times the probability of that event. All of these condition on the qth-highest signal of the other bidders being x and the signal of i being x. This is the expected value of the $q + 1$st valuation conditional on $X_i = x$ and $Y_q = x$. Solving this first-order differential equation yields the expression for the bidding function. The monotonicity of $w(x, x)$ implies that the condition is sufficient as well. *Q.E.D.*

Proof of Theorem 3. Let $G(\cdot)$ denote the distribution function of the bidder signals, X_i, and $g(\cdot)$ be its associated density. The expected value of the sum of the top q bids is

$$\sum_{i=1}^{q} E[B_q(X_{(i)})] = \int_0^1 B_q(x)\sum_{i=1}^{q} n\binom{n-1}{i-1}(1-G(x))^{i-1}G^{n-i}(x)g(x)\,dx$$

$$= \int_0^1 B_q(x)\sum_{j=0}^{q-1}\binom{n-1}{j}(1-G(x))^{j}G^{n-1-j}(x)ng(x)\,dx.$$

By the independence of signals,

$$K_{Y_q}(x) = \sum_{j=0}^{q-1}\binom{n-1}{j}(1-G(x))^{j}G^{n-1-j}(x).$$

So, substituting the definition of $B_q(X)$,

$$\sum_{i=1}^{q} E[B_q(X_{(i)})] = \int_0^1\left(\int_0^x w(y, y)k_{Y_{(q)}}(y)\,dy\right)ng(x)\,dx.$$

Integrating by parts gives

$$\sum_{i=1}^{q} E[B_q(X_{(i)})] = \int_0^1 w(x, x)k_{Y_{(q)}}(x)n(1-G(x))\,dx = q\int_0^1 w(x, x)n\binom{n-1}{q}(1-G(x))^{q}G^{n-1-q}(x)g(x)\,dx$$

$$= q{*}E[w(X_{(q+1)}, X_{(q+1)})].$$

Q.E.D.

Proof of Theorem 4. Theorems 1 and 3 yield the result. *Q.E.D.*

Proof of Theorem 5. As m becomes large, (A1) in the proof of Theorem 2 approaches

$$B_q(x)k_{Y_{(q)}}(x) + \frac{dB_q(x)}{dx}K_{Y_{(q)}}(x) = k_{Y_{(q)}}(x)\left\{\int_0^1 u_i\,dF_{U_i}(u_i|X_i=x)\right\}.$$

This is the first-order differential equation defining the standard bidding function for an independent private-values, q-object auction. Revenue equivalence implies that expected revenue approaches the expected private value of the $q + 1$st-highest license valuation, the first equality in the statement of the theorem. Let $B_{q-1}(x)$ be a bidding function in a $q - 1$-unit auction. Applying the same argument to this auction yields

$$\sum_{i=1}^{q-1} E[B_{q-1}(X_{(i)})] = (q - 1)E[U_{(q)}].$$

Furthermore,

$$\lim_{m\to\infty} B_{q-1}(x) \geq \lim_{m\to\infty} B_q(x), \quad \forall x.$$

Combining these results yields the conclusions of the theorem. *Q.E.D.*

Appendix B

■ This Appendix shows that allowing bidders to purchase speculatively at the auction creates common-value-like effects in the auction that can suppress bidding behavior.

The first equation in the proof of Theorem 2 can be rewritten as (replacing $Y_{(q)}$ with $Y_{(s)}$)

$$\int_0^1\int_0^1\int_0^{u_i}(u_i - v_{(q)})\, dF_{v_{(q)}}(v_{(q)}|Y_{(s)} = y)\, dF_{U_i}(U_i|x)k_{Y_{(s)}}(y)\, dy$$

$$+ \int_0^{\tilde{x}}\left\{\int_0^1\int_{u_i}^1 v_{(q+1)}\, dF_{V_{(q+1)}}(v_{(q+1)}|Y_{(s)}= x)\, dF_{U_i}(u_i|X_i = x) + \int_0^1\int_0^{u_i} v_{(q)}\, dF_{V_{(q)}}(v_{(q)}|Y_{(s)} = x)\, dF_{U_i}(u_i|X_i = x)\right.$$

$$\left. + \int_0^1\int_{u_i}^1\int_0^{u_i} u_i\, dF_{V_{(q+1)}}(v_{(q+1)}|Y_{(s)} = x, V_{(q)} = v_{(q)})\, dF_{V_{(q)}}(v_{(q)}|Y_{(s)} = x)\, dF_{U_i}(u_i|X_i = x) - B_q(\tilde{x})\right\}k_{Y_{(s)}}(y)\, dy.$$

This can be written simply as

$$[u_i - v_{(q)}|x, v_{(q)} \leq u_i]*\text{Prob}[v_{(q)} \leq u_i|x] + E[U_{(q+1)}|Y_{(s)} \leq \tilde{x}, x]*\text{Prob}[Y_{(s)} \leq \tilde{x}|x] - B_q(\tilde{x})*\text{Prob}[Y_{(s)} \leq \tilde{x}|x]. \tag{B1}$$

The first term is independent of $\bar{x}$ and therefore independent of the bid. The last two terms are the same as the return to a bidder with signal x, who bids as if he had signal $\tilde{x}$, who wins whenever $\tilde{x}$ is higher than the s-highest signals and whose only return is the payoff from selling the quota at a price equal to the $q + 1$st-highest valuation. (That is, the bidder has no use value for the object.)

Suppose that bidders can submit multiple-unit bids up to a maximum of f. Let $q/f = s$ and assume that s is a whole number. This latter assumption helps to ensure symmetry among bidders. Redefine $w(\cdot, \cdot)$ such that

$$w(x, y) = E[U_{(q+1)}|X_i = x, Y_{(s)} = y].$$

Define a per-unit bidding function as

$$B_q(x) = E[w(Y_{(s)}, Y_{(s)})|Y_{(s)} \leq x].$$

Fix a bidder i and assume that for each of the other $n - 1$ bidders, if the bidder's signal is z, he submits a bid $B_q(z)$ for each of the f objects he is allowed to bid for. Consider bidder i's bid, b, for any single object holding the bid for the other $f - 1$ objects fixed. If $b < B_q(Y_{(s)})$, then the bid will not win a license. If $b > B_q(Y_{(s)})$, then there are at most $(s - 1)*f + (f - 1) < q$ bids above b and the bid will win a license. Thus, b wins if and only if $b < B_q(Y_{(s)})$ and, in particular, wins independent of bids the same bidder submits on the other $f - 1$ licenses.

Therefore, whether or not the bidder has a use value for the object, a necessary and sufficient condition for this bid to be optimal is that it maximize the value of (B1). Using the same argument as in the proof of Theorem 2, bidding according to the revised definition of B_q for each object solves that maximization problem. The remainder of the arguments in Theorems 3 and 4 remain true, replacing $Y_{(q)}$ by $Y_{(s)}$. However, Theorem

5 will no longer be applicable, since (1) does not imply the probability that $U_{(q)} < U_i$ given $Y_{(s)} > X_i$ will go to zero. In fact, it will generally not be zero, and this lowers bids relative to the prediction in Theorem 5.

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