

Endogenous Availability, Cartels, and Merger in an Equilibrium Price Dispersion

R. PRESTON McAFEE*

*Department of Economics, University of Texas at Austin,
Austin, Texas 78712*

Received October 10, 1991; revised September 23, 1992

In a generalization of the Butters advertising model, the equilibrium involves one firm that advertises at least twice as much as the others, who all advertise equally. A cartel formation game is considered, and any equilibrium cartel involves at least two, but generally not all, firms. In one equilibrium, only the largest firms join the cartel. A merger game is considered, and, in equilibrium, the large firm buys the other firms in sequence, with discounting equalizing the expected utility of the targets and prices rising over time. *Journal of Economic Literature Classification Numbers: D43, D83, L11, L13, L41.* © 1994 Academic Press, Inc.

The equilibrium price dispersion model of Butters [2]¹ is perhaps the most parsimonious price dispersion model. In this model, n identical firms send out price offers for a homogeneous good, and the consumers receive the offers according to a binomial distribution. An important feature of this model is that there is a unique equilibrium, which involves firms using a mixed strategy over prices, and that the distribution of prices has a simple closed form.

I address two types of questions concerning this model. First, what are the effects of endogenously choosing the level of advertising? And, second, what are the incentives governing cartel formation and mergers in this model?

In order to address the first question, it is necessary to consider the equilibrium distribution of prices when the probability that a consumer obtains a particular firm's price offer (which I call *the availability rate*) varies across firms. In addition to this generalization, I allow for downward sloping

* The thoughtful assistance of an anonymous referee and Deng-Yang Chou is gratefully acknowledged.

¹ Butters only considers the limiting case of infinitely many firms that advertise at an infinitesimal rate. He does consider increases in the advertising rate in this limiting case. Two closely related papers are Varian [8] and Burdett and Judd [1]. Varian posits consumers that either are informed about all prices or choose one price at random. Burdett and Judd consider various types of search equilibria, but only consider symmetric strategies by firms. See Carlson and McAfee [3] for a discussion of equilibrium price dispersion models.

demand, generalizing the unit demand considered by Butters. This does not turn out to introduce any complexity into the model, provided consumer search is prohibitively costly. Given the timing of events in the model, the notion of advertising employed in this paper corresponds more to the limited availability of goods, rather than pure price advertising.

I show that the profit levels per consumer contacted are identical among firms and depend on the availability rates of all but the largest firm and on the profits at the monopoly price. This is interesting because it introduces an asymmetry into the profit expressions, and, in particular, the profits vary in availability rates differently for the largest (most widely available) firm and other firms. As a result, when firms can choose their availability rates, there will tend to be one large firm and $n - 1$ smaller firms. That is, the symmetric availability rates assumed by Butters will not be obtained when availability is an endogenous variable.

Firms in this model have an incentive to coordinate their activities, in this case, to offer the same price. I provide two results about cartels in this framework. First, one equilibrium cartel involves only large firms. That is, there will be an integer k so that the k largest firms join the cartel and no smaller firms join. To my knowledge, this is the first result of its kind and can be used to justify an antitrust analysis based on concentration ratios, for some markets, because the firms involved in a cartel will be large firms.² Second, $k \geq 2$, that is, the cartel invariably involves the two largest firms. By example, it can be shown that k can be anything from two to the entire industry.

The analysis of mergers is different from the cartel analysis. It turns out that the largest firm always has an incentive to purchase at least one of its rivals at a cost equal to what the rival's profits would have been in the absence of a takeover. For example, suppose that there is one large firm and n smaller firms, and let π_k denote the profits to a small firm when there are k small firms and $n - k$ small firms merged with the large firm. Then the large firm will always be willing to pay π_k for an additional firm when there are k left. In essence, the large firms are willing to "walk up" the supply curve and buy the entire industry. However, the large firm is not willing to pay π_1 for all of the small firms, which is what the small firms would hold out for, provided they expect the large firm to merge to monopoly.

An equilibrium resolution to this consistency problem involves the large firm buying other firms over time, so that the higher prices paid for the later acquisitions are discounted by the acquired firms, thus accounting for why some sell out at initially lower prices. In the limiting case of a continuum of infinitesimal firms, it turns out that the acquiring firm pays

² However, basing an antitrust policy on deterrence of cartels is probably ill-advised; see McAfee and Williams [5].

his marginal value of the target to the target and that this increases over time. Higher discounting works in the favor of the acquirer and speeds the acquisition process up. The acquirer grows at a continuous rate until a certain size is reached, at which time the acquirer acquires the rest of the industry at a common price.

The structure of the paper is as follows. First, the equilibrium profits are examined when a proportion $\alpha, \in (0, 1)$ of the consumers see firm i 's price. The main body of the paper concentrates exclusively on the case where consumer search is prohibitively expensive. However, in Appendix B, I show that many of the results extend to the case where consumers can search. The choice of availability rates is endogenized in the third section, so that the firms first choose α_i , then these become known, and then the firms choose their prices. I then consider a game where firms have exogenously specified availability rates, choose whether or not to join a cartel, and then play a pricing game. Finally, a related merger game is considered. Because I employ subgame perfection as my equilibrium notion, the analysis in all cases begins with the pricing subgame. The paper ends with a conclusion.

The model is closely related to that of Robert and Stahl [7] (R-S). The main distinction is that R-S have firms choose their availability rates simultaneously with the choice of prices. In R-S, lower priced firms have more widely available or heavily advertised goods. This is a qualitatively different outcome, for in the present study it turns out that the firm with wider availability has higher prices, in the sense of first-order stochastic dominance, than the firms with less market presence. In addition, R-S find a symmetric equilibrium, where the present study finds an asymmetric equilibrium from an initially symmetric model. The present model is more appropriate for some environments and not for others.

The timing of the model can be motivated by the following story. There are a number of malls, each possessing a large number of stores. Firms can choose to market their products in a proportion of these stores. However, getting a store to carry a product requires paying a "shelf-space" cost per store that carries the product, and the firms cannot influence which stores are located in which malls, because the product the firm sells is a small proportion of the store's product line. Thus, adjustment of the number of stores carrying the firm's product is slow, while the ability of the firms to adjust prices is nearly continuous, justifying the assumption that the firms choose prices knowing the availability rates, or the proportion of stores carrying the product, of all the firms, but not knowing the current prevailing prices of the other firms. Consumers shop in the mall nearest to them,³ and sample all the stores in that mall for free. It is prohibitively

³ Alternatively, consumers could call all of the local stores that might carry the product to obtain price quotes, rather than visiting a mall.

expensive to visit a second mall, perhaps because the good in question represents a small fraction of the consumer's purchases on a given trip to the mall. I, however, consider a special case where a consumer can visit a second mall at some cost in Appendix B. The R-S model is, in contrast, more appropriate for newspaper advertisement and other "one-time" advertising decisions, that is, it corresponds more closely to price advertising than goods availability.

In this model, some consumers will receive no price quotes. This leaves a residual demand. I allocate this residual demand as follows. In the main model, these consumers do not purchase, corresponding to a prohibitive search cost. The case of unit demand, allowing for search, is developed in Appendix B. At cost σ , any consumer can visit a second mall and receive a second draw on the distribution of prices, a draw which is independent of the first. This has two main effects. First, provided that σ is not too large, all consumers obtain the good. Second, there is a reservation price r , so that, if a consumer observes a best price higher than the reservation price, search is induced. Obviously no firm will find it profitable to price above this reservation price. The main point of Appendix B is that the conclusions of the model without search are robust to consumer search.

THE PRICING STAGE

There are n firms, with firm i having an availability rate α_i , which means that a given consumer receives firm i 's offer with probability α_i , which is independent of reception of offers from other firms. I index the firms so that

$$1 > \alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n > 0.$$

Let F_i be the cumulative distribution function of firm i 's offered prices. Consider a consumer who receives at least one price offer. This consumer will buy from the lowest price firm (randomizing equally in the event of a tie) and will buy $q(p)$, where p is the lowest price observed by this consumer. This leads to profits of

$$R(p) = (p - mc) q(p)$$

for the low-priced firm, where mc is the marginal cost of production, which is assumed constant. I assume that R takes a maximum at $p^m \in (mc, \infty)$ and that R is increasing on (mc, p^m) . Consumers will be forbidden to search.

Denote the support of prices offered by firm i by S_i , and let L_i, H_i be the lowest and highest prices in the closure of the support:

$$L_i = \inf(S_i),$$

$$H_i = \sup(S_i).$$

The following four facts are straightforward to establish; I provide only a sketch of the argument.

$$\max_i \{H_i\} \leq p^m. \quad (1)$$

A firm that prices above the monopoly price can lower price to p^m , thereby increasing both the probability of having the low price and the profits conditional on the low price.

$$F_i \text{ is continuous on } [L_i, p^m). \quad (2)$$

Suppose F_i has a jump at $p \in [L_i, p^m)$. Then there is an $\varepsilon > 0$ so that, for $j \neq i$, $F_j(p) = F_j(p + \varepsilon)$, since firm j gets a discrete increase in the probability of having the low price by pricing at $p - \delta$ instead of $p + \varepsilon$, for arbitrarily small δ . But this means that firm i can raise its price to $p + \varepsilon$ without loss in the number of consumers, contradicting the optimality of pricing at p .

$$\max_i \{H_i\} = p^m. \quad (3)$$

Suppose all firms price below $H_m < p^m$. Consider the firm offering the highest price offered, H_m . This firm can raise price to p^m without decrease in the probability of offering the lowest price, which is $\prod_{i \neq m} (1 - \alpha_i)$.

$$\text{At most one firm has an atom at } p^m. \quad (4)$$

Otherwise, each of the firms with an atom at p^m has an incentive to lower the price below p^m . Facts (2) and (4) make the probability distributions continuous, except for possibly one firm at the maximum of the support. Therefore, firm i 's profits can be written so that, for $p < p^m$,

$$(\forall p \in S_i) \pi_i = \alpha_i R(p) \prod_{j \neq i} (1 - \alpha_j F_j(p)). \quad (5)$$

Let $L = \min_i \{L_i\}$, the lowest price offered by any firm. If a firm has an atom in its price distribution at p^m , call this firm m , otherwise let firm m be a firm with $H_m = p^m$, which must exist by (3).

LEMMA 1. For all i , $L_i = L$, which satisfies

$$R(L) = R(p^m) \prod_{j \neq m} (1 - \alpha_j).$$

Moreover, $m = 1$.

Proofs are provided in Appendix A.

From Lemma 1, I can immediately deduce that all firms earn the same expected profits per consumer contacted, which gives profits of

$$\pi_i = \alpha_i R(L) = \alpha_i R(p^m) \prod_{j \neq i} (1 - \alpha_j). \tag{6}$$

Remark 1. The firm with the highest availability rate has no effect on the equilibrium profits of the other firms. It is an interesting feature of this model that profits depend on demand through the profits at the monopoly price. Therefore, there is no difference in profits between different demand curves with the same monopoly profits. As is shown below, however, the distribution of prices depends on the demand curve.

Lemma 2 shows that there exists an equilibrium. Interpret H_{n+1} to be L .

LEMMA 2. *There exists an equilibrium, with $p^m = H_1 = H_2 \geq H_3 \geq \dots \geq H_n$ satisfying*

$$\frac{R(H_i)}{R(p^m)} = \prod_{j=2}^i \left(\frac{1 - \alpha_j}{1 - \alpha_i} \right), \tag{7}$$

and, for $k \leq i$, and $p \in (H_{i+1}, H_i]$, $i = 2, \dots, n$

$$1 - \alpha_k F_k(p) = \left(\frac{R(p^m)}{R(p)} \prod_{j=2}^i (1 - \alpha_j) \right)^{(1/(\alpha_i - 1))} = (1 - \alpha_i) \left(\frac{R(H_i)}{R(p)} \right)^{(1/(\alpha_i - 1))}. \tag{8}$$

Finally, $\lim_{p \rightarrow p^m} F_1(p) = F_1(p^m) = \alpha_2/\alpha_1$.

While I am uncertain whether this equilibrium is unique, it is the only equilibrium where firms employ interval support of prices. Moreover, the profits are the same in any equilibrium, and the use of this pricing equilibrium for the present study is confined to determining the payoffs to various actions, such as the choice of availability rates or cartel formation, which depend only on the profits in the pricing subgame, which are the same for all equilibria. It follows from (8) that firms which have high availability rates employ higher prices, in contrast to the model of Robert and Stahl [7]. There is a simple intuition for this observation, provided by a referee. A consumer receives an offer from only firm i with probability $\alpha_i \prod_{j \neq i} (1 - \alpha_j)$. This is clearly nonincreasing in i , that is, the firms with the higher availability rates are more likely to represent a consumer's only offer and therefore have an incentive to charge higher prices. Casual empiricism indicates that often the firms that have high prices have wider availability or market presence. However, this observation confounds the fact that brand names tend to be advertised more heavily and cost more than

generic products, so the observation does not offer real support for the model. It is notable that Lemma 2 implies there is a mass point at the maximum of the distribution if the largest firm has wider availability than the other firms.

The R-S model finds that firms with high α 's charge lower prices because firms choose availability and prices simultaneously. Thus, at the time a firm chooses its price, its expectation about the likelihood of facing competition is invariant to its choice of availability. Thus the main driving effect in the present study, that firms with relatively high availability face a lower probability of competition, is absent in the R-S model, because R-S firms pick prices before knowing the realization of the extent of competition. Finally, having chosen a low price, an R-S firm has a higher return to more advertising, because it is more likely to "win" any given consumer, inducing the negative correlation between prices and advertising that they find. Firms with high prices will mostly get demand only from sequential searchers, that is, their advertisements are less likely to increase their demand.

The Selection of Availability Rates

Let $c(\alpha)$ be the cost of availability rate α . I assume that c is twice continuously differentiable, strictly increasing and strictly convex on $(0, 1)$. In addition, I further assume that

$$c(0) = 0, \quad (9)$$

$$c'(0) < R(p^m), \quad (10)$$

and

$$\lim_{\alpha \rightarrow 1} c'(\alpha) > R(p^m). \quad (11)$$

Equation (9) eliminates fixed costs, which play no role in this section since n is exogenous. Equation (10) ensures that a monopoly will choose positive α , while (11) rules out $\alpha = 1$. Firms simultaneously choose their availability rates and then play the pricing game of the previous section. I continue to index the firms so that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$. Up to the permutation of firms across indices, there is a unique pure strategy equilibrium to this stage of the game.

THEOREM 3. *In any pure strategy equilibrium, $\alpha_1 > \alpha_2 = \alpha_3 = \dots = \alpha_n$. The values of α_1 and α_2 are given by*

$$c'(\alpha_2) = (1 - 2\alpha_2) R(p^m)(1 - \alpha_2)^{n-2}, \quad (12)$$

and

$$c'(\alpha_1) = R(p^m)(1 - \alpha_2)^{n-1}. \tag{13}$$

There exists a pure strategy equilibrium, which is unique up to the identity of firm 1. Moreover, firm 1 earns strictly higher profits than firms 2, ..., n.

Remark 2. Provided $n \geq 2$, $\alpha_2 < \frac{1}{2}$.

It is not too surprising that initially identical firms exhibit different behavior in this model, because the model also generates random pricing behavior. There is, of course, a symmetric mixed strategy equilibrium, which is not analyzed here. The previous literature has focused on the symmetric pricing game, where all firms have the same availability rate. Theorem 3 indicates that this is not justified under remarkably weak conditions on the cost of availability.

Remark 3. It is straightforward to show that α_1 and α_2 decrease in n . The value of n could be uniquely determined by imposing a fixed cost of being a firm. However, this seems to provide no interesting economic insight.

Theorem 3 appears to be the most that can be said about the α_i without imposing further structure on c . To do so, consider the effects of merger in this model on costs. Consider two firms with availability rates α and β , respectively. By the assumed independence, these two firms reach a proportion $1 - (1 - \alpha)(1 - \beta)$ of the consumers, at cost $c(\alpha) + c(\beta)$. This should be the maximum that it costs a single firm to reach $1 - (1 - \alpha)(1 - \beta)$ of the consumers, provided that it can duplicate the process by which two firms reach this many consumers. Thus, the scale economy provided by combining operations is $c(\alpha) + c(\beta) - c(1 - (1 - \alpha)(1 - \beta))$. Consequently, I define *increasing (constant, decreasing) returns to scale* in availability to mean that $c(\alpha) + c(\beta) - c(1 - (1 - \alpha)(1 - \beta))$ is increasing (constant, decreasing) in α, β . The following lemma simplifies this expression.

LEMMA 4. c displays increasing (constant, decreasing) returns to scale if $(1 - \alpha) c'(\alpha)$ is decreasing (constant, increasing).

Remark 4. The case of constant returns to scale implies a particular functional form for c ,

$$c(\alpha) = -\theta \ln(1 - \alpha),$$

where $\theta > 0$. The Butters [2] model obtains in the limit as the number n of firms diverges, with $q(p)$ equal to one if $p \leq p^m$, and zero otherwise.

The notion of returns to scale is a powerful tool for relating α_1 to α_2 .

LEMMA 5. If c displays increasing (constant, decreasing) returns to scale, then $\alpha_1 > (=, <) 2\alpha_2$.

This model predicts a particular industry structure, which involves a natural "leader" (although not in the Stackelberg sense, of course), with one firm twice as large as the others, in the case of constant returns to scale in availability.

Remark 5. The proportion of the market which receives an offer is $1 - (1 - \alpha_1)(1 - \alpha_2)^{n-1} = 1 - (1 - \alpha_1)c'(\alpha_1)/R(p^m)$. Thus, an increase in n leads to fewer consumers being contacted, if $(1 - \alpha)c'(\alpha)$ is decreasing, since α_1 decreases in n . That is, if c displays increasing returns to scale, an increase in n will lead to fewer consumers being reached. This is intuitive, because an increase in n means that economies of scale will be exploited less fully. This has an interesting consequence when consumers have unit demand at any price not exceeding p^m . In this case, the net social surplus of the pricing game falls as the number of firms increases.

A Cartel Game

Suppose the firms have availability rates $1 > \alpha_1 \geq \dots \geq \alpha_n > 0$. The largest firm, firm 1, simultaneously asks the other firms if they would like to form a cartel.⁴ Firms may respond yes or no; those that respond yes join the cartel. If A is the set of cartel members, the cartel awards firm $i \in A$ with a share $\alpha_i / \sum_{j \in A} \alpha_j$ in the cartel profits. This shares the cartel profits proportionately to profits in the equilibrium without the cartel, since π_i is proportional to α_i . It is presumed that, before the pricing game takes place, the identities of cartel members become known. This ensures that the pricing game is a full information game. The cartel is assumed to maximize the sum of profits; that is, it acts like a single firm with probability $1 - \prod_{j \in A} (1 - \alpha_j)$ of contacting a particular consumer.⁵ I abstract away from enforcement issues in this cartel formation game, by assuming that the cartel has some method of ensuring that cartel members follow the cartel strategy and that the cartel has no fear of prosecution by the government.⁶

⁴ Having the largest firm ask the other firms ensures that the resulting cartel is at least as large as any cartel nonmember. Cartels that are smaller than the largest cartel nonmember earn strictly less than they would as independent firms.

⁵ This cartel game is related to the cartel formation game in McAfee and McMillan [6]. However, that game is cast in the context of bidding in auctions, and they do not consider asymmetric shares. It is a curious fact that the Butters pricing game is formally equivalent to a first price sealed bid auction, where bidders' values are drawn independently from $\{0, R(p^m)\}$.

⁶ This is not as unreasonable as it might appear. Until the 1987 sentencing guidelines came into effect, actual sentences served by convicted price-fixers were minimal, around 3 months in minimum security prisons known as "Club Fed." Moreover, international cartels are, of course, legal. Provided enforcement imposes a fixed penalty on a convicted cartel member, the qualitative results of this section are unchanged.

The effect of the cartel is to prevent the member firms from undercutting each other; thus, one view of the cartel is that it ensures that the member firms all offer the same price. Provided that the cartel does not involve the entire set of firms, the cartel will also randomize the price it offers.

A Nash equilibrium to the cartel game is defined by two properties. First, those in the cartel are better off in than out, and, second, those out of the cartel are better off out than in. There are two possible formulations of these properties, depending on whether membership in the cartel is observable to nonmembers. In what follows, I examine only the case where membership is observable, which is the easier of the two cases, because it makes the pricing game a subgame. Mathematically, the first property is

$$\begin{aligned}
 & (\forall i \in A) \alpha_i (1 - \alpha_i) R(p^m) \prod_{j \neq i} (1 - \alpha_j) \\
 & \leq \frac{\alpha_i R(p^m)}{\sum_{j \in A} \alpha_j} \left[1 - \prod_{j \in A} (1 - \alpha_j) \right] \prod_{j \neq i} (1 - \alpha_j). \quad (14)
 \end{aligned}$$

To see this, first note that the cartel is a single large firm, with availability rate $\alpha_A = 1 - \prod_{j \in A} (1 - \alpha_j)$. By (6), cartel profits are $\alpha_A R(p^m) \prod_{j \neq i} (1 - \alpha_j)$. Thus, the right hand side of (14) is just i 's share of the cartel profits times the amount of the cartel profits. If i does not join the cartel, there will be one extra firm not participating in the cartel, and since the cartel always contains firm 1, the cartel is the largest firm. Thus, the left hand side of (14) gives i 's payoff if he fails to join the cartel.

Similarly, a firm that does not join the cartel in equilibrium must make at least as large profits from staying out as from joining. Thus,

$$\begin{aligned}
 & (\forall i \notin A) \alpha_i \prod_{j \neq i} (1 - \alpha_j) \\
 & \geq \frac{\alpha_i}{\alpha_i + \sum_{j \in A} \alpha_j} \left[1 - (1 - \alpha_i) \prod_{j \in A} (1 - \alpha_j) \right] \prod_{\substack{j \neq A \\ j \neq i}} (1 - \alpha_j). \quad (15)
 \end{aligned}$$

The interpretation of (15) is similar to the interpretation of (14).

THEOREM 6. *There exists an equilibrium to the cartel game with $A = \{1, \dots, k\}$, and $k \geq 2$. The value of k is given by:*

$$(1 - \alpha_k) \sum_{j=1}^k \alpha_j + \prod_{j=1}^k (1 - \alpha_j) \leq 1 < (1 - \alpha_{k+1}) \sum_{j=1}^{k+1} \alpha_j + \prod_{j=1}^{k+1} (1 - \alpha_j). \quad (16)$$

For any equilibrium cartel A , let $i \in A$ and $l \notin A$. Then

$$\frac{\alpha_i}{\alpha_l} \geq \frac{\alpha_l}{1 - \alpha_l + \alpha_l^2}.$$

Remark 6. There are two main conclusions from Theorem 6. First, the cartel is always nontrivial, because two firms always join it. In addition, it can be shown that, if $\alpha_1 = \alpha_2 = \alpha_3$, then $k \geq 3$. This is the largest lower bound that holds independently of the α_i levels. For α_4 sufficiently small but positive, $k = 3$, and $k = n$, the cartel of all firms, if α_n is close enough to 1. Second, there is an equilibrium cartel involving only the largest firms. This seems to me like a reasonable property for a cartel model to possess, but, to my knowledge, this is the first model with the property that there is an equilibrium cartel composed only of large, and not small, firms. While the equilibrium is not generally unique, the inequality shows that a cartel cannot exclude very large firms when it includes small ones. In particular, if the cartel excludes a firm with availability rate $\frac{1}{2}$, it cannot include firms with rates less than $\frac{1}{3}$. Thus, even when there are equilibria excluding some large firms, no very small firms are included.

If any firms remain outside the cartel, the cartel does not exploit the full profit potential of the market. It would be logical for the cartel to attempt to share the profits in such a way that all firms are induced to join the cartel. Let s_i be the share of cartel profits accruing to firm i . Firm i will wish to join the cartel provided:

$$\alpha_i(1 - \alpha_i) R(p^m) \leq s_i \left(1 - \prod_{j=1}^n (1 - \alpha_j) \right) R(p^m). \quad (17)$$

That is, the profits when firm i goes alone do not exceed firm i 's share of the cartel profits.⁷ This is not generally possible. Define a *feasible sharing rule* to satisfy, for any cartel A , $\sum_{j \in A} s_j \leq 1$. A sharing rule is feasible if it does not require subsidies from outside the cartel.

THEOREM 7. *For any feasible sharing rule, there are small positive availability rates that would induce some firms not to join the cartel, provided $n \geq 4$. For $n = 3$, the sharing rule*

$$s_i = \frac{\alpha_i(1 - \alpha_i)}{\sum_{j \in A} \alpha_j(1 - \alpha_j)}$$

leads to the cartel of all firms.

Until 1982, the U.S. Department of Justice used *concentration ratios* as the basis for evaluating the concentration in an industry. The k firm concentration ratio is the sum of the market shares of the k largest firms. The typical concentration ratio used was the four firm concentration ratio, or CR4. Theorem 7 suggests a motivation for this approach. First, cartels

⁷ I am assuming that $\alpha_i \leq \sum_{j \neq i} \alpha_j$.

may involve the k largest firms. Provided the top three are similar in size, k will be at least 3. This is in contrast to the Cournot model of Cramton and Palfrey [4], in which it is difficult to draw large firms into a cartel.

It would be useful to endogenize the levels of α in the cartel formation game, to find out if more can be said about the numbers of firms in and out of the cartel. However, there are serious difficulties with such an analysis. Even in the case with two firms, where we know from Theorem 6 that a cartel will form, the profit functions net of the cost of availability are poorly behaved in α . It is not generally clear that a pure strategy equilibrium exists even for the simple cost function given in Remark 4.

Creating a Monopoly

Suppose that entry of new firms is not possible, so that there is a fixed number of firms with fixed availability rates. Can the firms profitably merge to monopoly? First note that profitable mergers always “create a new largest firm,” either because the merger involves the largest firm or because the merged firm reaches more consumers than the old largest firm. To see this, let i and j merge, creating a firm with availability rate $\alpha_m = (1 - (1 - \alpha_i)(1 - \alpha_j)) < \alpha_1$. This firm earns profits of

$$\begin{aligned} \pi_m &= (1 - (1 - \alpha_i)(1 - \alpha_j)) \left(\prod_{l \neq 1, i, j} (1 - \alpha_l) \right) (1 - \alpha_m) \\ &< (\alpha_i + \alpha_j) \prod_{l \neq 1} (1 - \alpha_l) = \pi_i + \pi_j. \end{aligned}$$

If the merger involves firm 1, it invariably creates larger profits:

$$\pi_m = (1 - (1 - \alpha_i)(1 - \alpha_1)) \prod_{l \neq 1, i} (1 - \alpha_l) > (\alpha_i + \alpha_1) \prod_{l \neq 1} (1 - \alpha_l) = \pi_i + \pi_1.$$

This shows that an additional merger is profitable if it involves the largest firm. However, it does not follow that the largest firm would like to buy all competing firms at a given price. Consider the following merger game. The largest firm simultaneously makes offers p_i for the purchase of all other firms. Any firm accepting the offer is merged into the largest firm, while if they reject, they remain an independent firm, and then the pricing game is played by the remaining firms. It is easily seen that this is formally equivalent to the cartel game, with the largest firm purchasing other firms, instead of offering a share in the cartel. From Theorem 7, this does not generally result in a merger to monopoly.

There is a tension between the pairwise incentive to merge and the incomplete merger to monopoly in the simultaneous game. Consider a situation where the simultaneous game does not result in a merger to

monopoly. Once the first round of offers is made, there is an incentive to make a new round of offers, because there is a remaining profitable merger. That is, there is an incentive to renegotiate. If a second round is introduced, the price for a given size firm will be higher in the second round than in the first, so some firms that would accept in a single round game may reject in the first round to enjoy higher prices in the second.

The resolution of this tension involves delay with discounting. Why are some firms willing to accept lower prices in early stages, rather than wait for higher prices later? Because to not accept now would delay those higher prices and delay the increased profits.

I examine this question in a limiting case of the price dispersion model, one with a continuum of firms indexed on the interval $[0, 1]$. I let μ be the proportion of these firms held in single ownership by a firm I call the conglomerate. Let $\pi(\mu)$ be the profits per unit of mass of the independent firms and $\gamma(\mu)$ be the profits of the large firm, which collectively holds μ . Assuming all firms have the same availability rate α and that the number of firms is sent to infinity in such a way that $\alpha n \rightarrow \beta$, these values will be

$$\pi(\mu) = \beta e^{-\beta(1-\mu)},$$

and,

$$\gamma(\mu) = \mu \beta e^{-\beta(1-\mu)}.$$

However, explicit functional forms are not necessary for this analysis, and only a few general properties are used. The first is that mergers are profitable, that is, $\gamma(\mu + \varepsilon) - \gamma(\mu) > \varepsilon \pi(\mu)$ for small ε , or,

$$\gamma'(\mu) > \pi(\mu). \quad (18)$$

I also assume γ is increasing and convex and π is nondecreasing.

A subgame perfect stationary equilibrium is a pair of functions $p: [0, 1] \rightarrow \mathbb{R}$ and $\mu: [0, \infty) \rightarrow [0, 1]$ so that the conglomerate will pay $p(\mu)$ for additional firms, if the conglomerate's mass is μ , and, in the interval of time $[t, s]$, the conglomerate purchases $\mu(s) - \mu(t)$ of the remaining independent firms. Note that stationarity and perfection are incorporated into this construction by presuming that, no matter how the conglomerate reaches the stage of owning μ of the firms, the amount it is willing to pay is $p(\mu)$. The equilibrium conditions are that the conglomerate optimizes by paying $p(\mu)$, when it owns μ , given that it will pay $p(\mu_0)$ for all $\mu_0 > \mu$ and given the behavior of the independent firms, and that the independent firms cannot do better than accept the conglomerate's offer. A useful definition is μ^* satisfying $\gamma'(\mu^*) = \pi(1)$. The following theorem displays a subgame perfect stationary equilibrium, when the discount rate is r .

THEOREM 8. *There exists a subgame perfect stationary equilibrium with*

$$p(\mu) = \frac{1}{r} \min\{\gamma'(\mu), \pi(1)\},$$

and

$$\mu'(t) = r \frac{\gamma'(\mu(t)) - \pi(\mu(t))}{\gamma''(\mu(t))}, \quad \text{for } \mu(t) < \mu^*.$$

If this leads to $\mu(T) = \mu^$, then the conglomerate buys all remaining firms at time T , at a price of $\pi(1)/r$.*

This is a very intuitive theorem. The price that the conglomerate pays leaves the conglomerate instantaneously indifferent between buying more or fewer firms, because the conglomerate pays its instantaneous marginal present value, $\gamma'(\mu)/r$, for additional firms. This leads the conglomerate to always be willing to buy as many firms as supply themselves at that price. Were the prices anything other than the conglomerate's instantaneous value, the conglomerate would have an incentive to buy either more or fewer of the firms and adjust prices accordingly. The number of independent firms supplied at that price is chosen so that the firms all have the same present value, no matter when they sell. At some point, prices reach the present value of being the only independent firm in the industry (when $\mu(T) = \mu^*$), and the rest of the firms sell out.

Because of the starkness of the model, the only meaningful comparative static that can be applied is a change in the discount rate. An increase in discounting effectively speeds up time, so that the acquisition rate increases. The amount of discounting that goes on between the conglomerate owning μ of the firms and owning μ_0 of the firms is invariant to the discount rate; otherwise the firms would not be indifferent. Increasing the discount rate works in the favor of the conglomerate by speeding up the acquisition process, as the value to independent firms of waiting for future higher prices is reduced.

For the particular functional forms suggested by the equilibrium price dispersion model, μ' is increasing. It is of some interest that the conglomerate pays its marginal value, despite the fact that I have assigned all of the "bargaining power" to the conglomerate, since only the conglomerate makes offers. Still, the conglomerate does not pay the ultimate marginal value of the firm, or even of the firm's ultimate value as an independent, for eventually a firm that remains independent will be worth $\pi(1)/r$, which is the most that the conglomerate ever pays. The discounting of the higher prices induces some firms to sell out earlier.

Unfortunately, this simple intuition does not extend to the finite number of firms case. There is an equilibrium to the symmetric finite case, which has the following form. If the conglomerate has purchased k firms, there is a prevailing price p_k for the purchase of another firm. The remaining independents choose a Poisson hit rate λ_k , for accepting the offer. The value of λ_k is such that the firms are indifferent to accepting now or not accepting and waiting for someone else to accept. The finite case differs in two significant ways. First, a holdout in the continuum case does not change the time path of acquisitions, where a holdout in the finite case expects a slower path of acquisitions and therefore smaller profits along the path. Second, if the price is slightly less than the conglomerate's value of an additional firm, then the conglomerate may not wish to increase its price to increase λ_k , because of the discrete effect of a single purchase. However, it appears from the continuum case that the price will be approximately the conglomerate's value of another acquisition.

Conclusion

The analysis of entry into the Butters [2] model indicates that the assumption of symmetry is not justified. Moreover, the prediction is that there will be one large firm, and a number of smaller, identical firms. The large firm will be at least twice as large as any smaller firm, at least when an economy of scale exists. In such a world, cartels will often not involve all firms, but only the largest firms. This is in accord with the stylized facts and provides a weak basis for considering concentration ratios. Finally, this paper gives a rationale for profitable mergers to be spread out over time and occur slowly. The analysis gives a preliminary reason for mergers to occur in waves, with the prediction that merger activity increases when discount rates are high.

I think the overall message of this paper is that the Butters model offers a sound alternative to the Cournot or Bertrand models for policy-oriented economic analysis. Although this paper focused on the positive predictions of the model, a normative analysis of the effects of policy instrument changes in the context of this model offers an exciting research agenda.

APPENDIX A: PROOFS

Proof of Lemma 1. Let $i \neq m$, and let p be a price in S_i . Then, from m 's optimization,

$$R(p^m) \prod_{j \neq m} (1 - \alpha_j) \geq R(p) \prod_{j \neq m} (1 - \alpha_j F_j(p)).$$

Similarly, from i 's optimization,

$$R(p) \prod_{j \neq i} (1 - \alpha_j F_j(p)) \geq R(p^m) \prod_{j \neq i} (1 - \alpha_j F_j^-(p^m)),$$

where $F_j^-(p^m) = \lim_{p \nearrow p^m} F_j(p)$, and for $j \neq m$, $F_j^-(p^m) = 1$, by (4).

Therefore,

$$\begin{aligned} R(p^m) \prod_{j \neq m} (1 - \alpha_j) &\geq R(p) \frac{1 - \alpha_i F_i(p)}{1 - \alpha_m F_m(p)} \prod_{j \neq i} (1 - \alpha_j F_j(p)) \\ &\geq R(p^m) \frac{1 - \alpha_i F_i(p)}{1 - \alpha_m F_m(p)} \prod_{j \neq i} (1 - \alpha_j F_j^-(p^m)), \end{aligned}$$

or

$$(1 - \alpha_i)(1 - \alpha_m F_m(p)) \geq (1 - \alpha_i F_i(p))(1 - \alpha_m F_m^-(p^m)).$$

In particular, for $p = L_i$,

$$1 - \alpha_i \geq (1 - \alpha_i)(1 - \alpha_m F_m(L_i)) \geq (1 - \alpha_m F_m^-(p^m)),$$

which forces $\alpha_m \geq \alpha_m F_m^-(p^m) \geq \alpha_i$ and hence $m = 1$.

Let $L_i \geq L_k = L$. Then, using first k 's optimization, then i 's optimization, we have

$$\begin{aligned} R(L) = R(L_k) &\geq R(L_i) \prod_{j \neq k} (1 - \alpha_j F_j(L_i)) \\ &= R(L_i) \prod_{j \neq i} (1 - \alpha_j F_j(L_i)) \frac{1 - \alpha_i F_i(L_i)}{1 - \alpha_k F_k(L_i)} \geq \frac{R(L)}{1 - \alpha_k F_k(L_i)}, \end{aligned}$$

which forces $F_k(L_i) = 0$ or $L_i \leq L$. Since L is the minimum possible value, we have $L_i = L$. Finally, L is given by m 's optimization:

$$R(L) = R(p^m) \prod_{j \neq 1} (1 - \alpha_j). \quad \blacksquare$$

Proof of Lemma 2. Let $p \in (H_{i+1}, H_i]$, and consider first $k \leq i$.

$$\begin{aligned} \pi_k &= \alpha_k R(p) \prod_{j \neq k} (1 - \alpha_j F_j(p)) \\ &= \alpha_k R(p) \prod_{j \neq k}^i (1 - \alpha_j F_j(p)) \prod_{j=i+1}^n (1 - \alpha_j) \\ &= \alpha_k R(p) (1 - \alpha_1 F_1(p))^{i-1} \prod_{j=i+1}^n (1 - \alpha_j) \\ &= \alpha_k R(p) \left(\frac{R(p^m)}{R(p)} \prod_{j=2}^i (1 - \alpha_j) \right) \prod_{j=i+1}^n (1 - \alpha_j) \\ &= \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j). \end{aligned}$$

Thus, the firm earns the same profits for any price in $[L_1, H_k]$. Now consider $k > i$, that is, a firm pricing above H_k .

$$\begin{aligned}
 \pi_k &= \alpha_k R(p) \prod_{j \neq k} (1 - \alpha_j F_j(p)) \\
 &= \alpha_k R(p) \prod_{i=1}^i (1 - \alpha_i F_i(p)) \prod_{\substack{j > i \\ j \neq k}}^n (1 - \alpha_j) \\
 &= \alpha_k R(p) (1 - \alpha_1 F_1(p))^i \prod_{\substack{j > i \\ j \neq k}}^n (1 - \alpha_j) \\
 &= \alpha_k R(p) \left(\frac{R(p^m)}{R(p)} \prod_{j=2}^i (1 - \alpha_j) \right)^{i/(i-1)} \prod_{\substack{j > i \\ j \neq k}}^n (1 - \alpha_j) \\
 &= \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j) \left(\frac{R(p^m)}{R(p)} \prod_{j=2}^i (1 - \alpha_j) \right)^{1/(i-1)} (1 - \alpha_k)^{-1} \\
 &\leq \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j) \left(\frac{R(p^m)}{R(H_{i+1})} \prod_{j=2}^i (1 - \alpha_j) \right)^{1/(i-1)} (1 - \alpha_k)^{-1} \\
 &= \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j) \left(\prod_{j=2}^{i+1} \frac{1 - \alpha_{j+1}}{1 - \alpha_j} \prod_{j=2}^i (1 - \alpha_j) \right)^{1/(i-1)} (1 - \alpha_k)^{-1} \\
 &= \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j) \frac{1 - \alpha_{i+1}}{1 - \alpha_k} \leq \alpha_k R(p^m) \prod_{j=2}^n (1 - \alpha_j).
 \end{aligned}$$

Thus, a firm that prices outside the support does worse than following the proposed equilibrium. That the distributions F_i are indeed distribution functions is a straightforward algebra exercise. Finally, observe that

$$\lim_{p \nearrow p^m} 1 - \alpha_1 F_1(p) = 1 - \alpha_2, \quad \text{so that } F_1(p^m) = \frac{\alpha_2}{\alpha_1}. \quad \blacksquare$$

Proof of Theorem 3. Profits of firm i are

$$\Pi_i = \pi_i - c(\alpha_i) = \alpha_i R(p^m) \prod_{j=2}^n (1 - \alpha_j) - c(\alpha_i).$$

Consider first the case of $\alpha_1 = \alpha_2$. Firm 1 must not wish to increase α_1 , so

$$0 \geq \frac{\partial \Pi_1}{\partial \alpha_1} = R(p^m) \prod_{j=2}^n (1 - \alpha_j) - c'(\alpha_1).$$

Firm 2 must not wish to decrease α_2 , so

$$0 \leq \frac{\partial \Pi_2}{\partial \alpha_2} = R(p^m)(1 - 2\alpha_2) \prod_{j=3}^n (1 - \alpha_j) - c'(\alpha_2).$$

These are contradictory unless $c'(0) = 0$, in which case $\alpha_1 = \alpha_2 = 0$, in which case $\alpha_i = 0$ for all i . But then $\partial \Pi_i / \partial \alpha_i > 0$, by (10). Therefore $\alpha_1 > \alpha_2$. For $i \geq 2$,

$$\frac{\partial \Pi_i}{\partial \alpha_i} = (1 - 2\alpha_i) R(p^m) \prod_{\substack{j=2 \\ j \neq i}}^n (1 - \alpha_j) - c'(\alpha_i).$$

Since $\alpha_i \geq \alpha_{i+1}$, $(1 - 2\alpha_i)/(1 - \alpha_i) \leq (1 - 2\alpha_{i+1})/(1 - \alpha_{i+1})$. Therefore $\alpha_i > \alpha_{i+1}$ implies $\partial \Pi_i / \partial \alpha_i < \partial \Pi_{i+1} / \partial \alpha_{i+1}$, and hence $\alpha_i = \alpha_{i+1}$. This establishes that $\alpha_1 > \alpha_2 = \alpha_n$. The first-order conditions give (12) and (13). It remains to be shown that this represents a subgame perfect equilibrium. First, consider firm 1. His profits are

$$\Pi_1(\alpha) = \begin{cases} \alpha R(p^m)(1 - \alpha_2)^{n-1} - c(\alpha) & \text{if } \alpha \geq \alpha_2 \\ \alpha(1 - \alpha) R(p^m)(1 - \alpha_2)^{n-2} - c(\alpha) & \text{if } \alpha < \alpha_2. \end{cases}$$

For $\alpha \leq \alpha_2$,

$$\begin{aligned} \Pi'_1(\alpha) &= (1 - 2\alpha) R(p^m)(1 - \alpha_2)^{n-2} - c'(\alpha) \\ &\geq (1 - 2\alpha_2) R(p^m)(1 - \alpha_2)^{n-2} - c'(\alpha_2) = 0. \end{aligned}$$

For $\alpha > \alpha_2$, Π_1 is strictly concave and therefore maximized at $\alpha = \alpha_1$.

Thus, firm 1 maximizes by choosing α_1 . Now consider firm $i \geq 2$.

$$\Pi_i(\alpha) = \begin{cases} \alpha(1 - \alpha) R(p^m)(1 - \alpha_2)^{n-2} - c(\alpha) & \text{if } \alpha \leq \alpha_1 \\ \alpha R(p^m)(1 - \alpha_2)^{n-2} (1 - \alpha_1) - c(\alpha) & \text{if } \alpha > \alpha_1. \end{cases}$$

For $\alpha < \alpha_1$, Π_i is concave and hence maximized at α_2 . For $\alpha > \alpha_1$,

$$\begin{aligned} \Pi'_i(\alpha) &= R(p^m)(1 - \alpha_2)^{n-2} (1 - \alpha_1) - c'(\alpha) \\ &\leq R(p^m)(1 - \alpha_2)^{n-2} (1 - \alpha_2) - c'(\alpha_1) = 0. \end{aligned}$$

Thus, Π_i is maximized at α_2 . Finally, firm 1 earns higher profits, as

$$\begin{aligned} \Pi_1 &= \alpha_1 R(p^m)(1 - \alpha_2)^{n-1} - c(\alpha_1) \\ &= \alpha_2 R(p^m)(1 - \alpha_2)^{n-1} - c(\alpha_2) + \int_{\alpha_2}^{\alpha_1} R(p^m)(1 - \alpha_2)^{n-1} - c'(s) ds \\ &> \alpha_2 R(p^m)(1 - \alpha_2)^{n-1} - c(\alpha_2) + \int_{\alpha_2}^{\alpha_1} R(p^m)(1 - \alpha_2)^{n-1} - c'(\alpha_1) ds \\ &= \alpha_2 R(p^m)(1 - \alpha_2)^{n-1} - c(\alpha_2) = \Pi_2. \quad \blacksquare \end{aligned}$$

Proof of Lemma 4. Let $\gamma = 1 - (1 - \alpha)(1 - \beta) \geq \beta$.

$$\begin{aligned} \frac{d}{d\beta} c(\alpha) + c(\beta) - c(1 - (1 - \alpha)(1 - \beta)) \\ = c'(\beta) - (1 - \alpha) c'(\gamma) \\ = (1 - \beta)^{-1} [(1 - \beta) c'(\beta) - (1 - \gamma) c'(\gamma)]. \end{aligned}$$

Thus $c(\alpha) + c(\beta) - c(1 - (1 - \alpha)(1 - \beta))$ is increasing (constant, decreasing) if

$$\beta \leq \gamma \text{ implies } (1 - \beta) c'(\beta) > (=, <) (1 - \gamma) c'(\gamma). \quad \blacksquare$$

Proof of Lemma 5. Note that $c'(\alpha_1) = ((1 - \alpha_2)/(1 - 2\alpha_2)) c'(\alpha_2)$. Therefore,

$$\alpha_1 \cong 2\alpha_2$$

as

$$1 - \alpha_1 \cong 1 - 2\alpha_2$$

as

$$(1 - \alpha_1) c'(\alpha_1) \cong (1 - 2\alpha_2) c'(\alpha_1) = (1 - \alpha_2) c'(\alpha_2).$$

The lemma follows from noting $\alpha_1 > \alpha_2$. \blacksquare

Proof of Theorem 6. From (14),

$$(\forall i \in A)(1 - \alpha_i) \sum_{j \in A} \alpha_j + \prod_{j \in A} (1 - \alpha_j) \leq 1. \quad (\text{A1})$$

From (15),

$$(\forall i \notin A)(1 - \alpha_i) \left(\alpha_i + \sum_{j \in A} \alpha_j \right) + (1 - \alpha_i) \prod_{j \in A} (1 - \alpha_j) \geq 1. \quad (\text{A2})$$

To see that $k \geq 2$, note that

$$(1 - \alpha_i)(\alpha_1 + \alpha_i) + (1 - \alpha_i)(1 - \alpha_i) = (1 - \alpha_i)(1 + \alpha_i) = 1 - \alpha_i^2 < 1.$$

Suppose that $A = \{1, \dots, k\}$ and that k satisfies (16). By (A1), cartel members satisfy (14). Define

$$\xi(\alpha) = (1 - \alpha) \left[\alpha + \sum_{j=1}^k \alpha_j + \prod_{j=1}^k (1 - \alpha_j) \right].$$

By (16), $\xi(\alpha_{k+1}) > 1$, so by (A2), (15) is satisfied for firm $k + 1$. Note that

$$\begin{aligned} \xi'(\alpha) &= -\alpha - \sum_{j=1}^k \alpha_j - \prod_{j=1}^k (1 - \alpha_j) + 1 - \alpha \\ &< 1 - \sum_{j=1}^k \alpha_j - \prod_{j=1}^k (1 - \alpha_j) \leq 0. \end{aligned}$$

Thus, for all $\alpha \leq \alpha_{k+1}$, $\xi(\alpha) \geq \xi(\alpha_{k+1}) > 1$, so by (A2), (15) is satisfied for all α_j , $i \geq k + 1$. Finally, let A be any set satisfying (14) and (15), $i \notin A$, $i \in A$. By (A2),

$$\begin{aligned} 1 &\leq (1 - \alpha_i) \left[\alpha_i + \sum_{j \in A} \alpha_j + \prod_{j \in A} (1 - \alpha_j) \right] \\ &\leq (1 - \alpha_i) \left[\alpha_i + \sum_{j \in A} \alpha_j + 1 - (1 - \alpha_i) \sum_{j \in A} \alpha_j \right] \\ &= 1 - \alpha_i^2 + (1 - \alpha_i) \alpha_i \sum_{j \in A} \alpha_j. \end{aligned}$$

The first inequality used (A2); the second used (A1). Also using (A1), we have

$$\alpha_i^2 \leq (1 - \alpha_i) \alpha_i \sum_{j \in A} \alpha_j \leq \frac{(1 - \alpha_i) \alpha_i}{1 - \alpha_i},$$

which rearranges to give the desired inequality. ■

Proof of Theorem 7. A feasible sharing rule satisfies $\sum_{i \in A} s_i \leq 1$. Thus, necessarily,

$$\sum_{i=1}^n \alpha_i (1 - \alpha_i) \leq 1 - \prod_{j=1}^n (1 - \alpha_j). \tag{A3}$$

Moreover, (A3) is sufficient, for if it is satisfied, the shares $s_i = (\alpha_i (1 - \alpha_i) / \sum_{j=1}^n \alpha_j (1 - \alpha_j))$ will satisfy (17). For $n \geq 4$, let $\xi(\alpha) = n\alpha(1 - \alpha) + (1 - \alpha)^n$. Note that

$$\begin{aligned} \xi'(\alpha) &= n[1 - 2\alpha - (1 - \alpha)^{n-1}], \\ \xi''(\alpha) &= n[-2 + (n - 1)(1 - \alpha)^{n-2}]. \end{aligned}$$

Thus $\xi(0) = 1$, $\xi'(0) = 0$, and $\xi''(0) = n(n - 3) > 0$. Therefore, for small α , $\xi(\alpha) > 1$, which contradicts (A3), with $\alpha_i = \alpha$, all i .

Now let $n = 3$, and $\alpha_1 \geq \alpha_2 \geq \alpha_3$, and

$$\begin{aligned}\mu(\alpha_1, \alpha_2, \alpha_3) &= \sum_{i=1}^3 \alpha_i(1 - \alpha_i) + \prod_{j=1}^3 (1 - \alpha_j), \\ \frac{\partial \mu}{\partial \alpha_1} &= 1 - 2\alpha_1 - (1 - \alpha_2)(1 - \alpha_3) \\ &\leq 1 - 2\alpha_1 - (1 - \alpha_1)^2 = -\alpha_1^2 < 0.\end{aligned}$$

Thus $\mu(\alpha_1, \alpha_2, \alpha_3) \leq \mu(\alpha_2, \alpha_2, \alpha_3)$.

$$\begin{aligned}\frac{d}{d\alpha_2} \mu(\alpha_2, \alpha_2, \alpha_3) &= 2(1 - 2\alpha_2) - 2(1 - \alpha_2)(1 - \alpha_3) \\ &\leq 2[1 - 2\alpha_2 - (1 - \alpha_2)^2] = -2\alpha_2^2 < 0.\end{aligned}$$

Thus, for $\alpha_1 \geq \alpha_2 \geq \alpha_3$,

$$\begin{aligned}\mu(\alpha_1, \alpha_2, \alpha_3) &\leq \mu(\alpha_2, \alpha_2, \alpha_3) \leq \mu(\alpha_3, \alpha_3, \alpha_3) \\ &= 3\alpha_3(1 - \alpha_3) + (1 - \alpha_3)^3 = 1 - \alpha_3^3 < 1,\end{aligned}$$

as desired. ■

Proof of Theorem 8. The conglomerate obtains

$$\int_0^{\infty} e^{-rt} [\gamma(\mu(t)) - p(\mu(t)) \mu'(t)] dt.$$

The conglomerate does not wish to deviate from p if a momentary increase in $\mu'(t)$ is not profitable, which leads to the euler equation

$$0 = e^{-rt} [\gamma'(\mu) - rp(\mu)].$$

However, a price of $\pi(1)/r$ is the most an independent firm can earn, so

$$p(\mu) = \frac{1}{r} \min\{\gamma'(\mu), \pi(1)\}.$$

A firm that sells out to the conglomerate at time t earns

$$\int_0^t e^{-rs} \pi(\mu(s)) ds + e^{-rt} p(\mu(t)).$$

In order to induce any firm to sell out, this expression must be constant with respect to t , which gives

$$0 = e^{-rt} [\pi(\mu(t)) - rp(\mu(t)) + p'(\mu(t)) \mu'(t)],$$

or

$$\mu'(t) = \frac{rp(\mu(t)) - \pi(\mu(t))}{p'(\mu(t))} = r \frac{\gamma'(\mu(t)) - \pi(\mu(t))}{\gamma''(\mu(t))}.$$

At time T , where $\mu(T) = \mu^*$, $p(\mu(t)) = \pi(1)/r$, and μ jumps to 1. ■

APPENDIX B: CONSUMER SEARCH

In this Appendix, I consider the case where there is unit demand, that is, $q(p) = 1$ if $p \leq \bar{p}$, and zero otherwise. However, consumers will be permitted to search, which gives them a new draw on prices at cost σ . I assume that σ is sufficiently low that a consumer who fails to obtain the good chooses to search. This has two effects on the model. First, consumers who fail to receive a price offer will search again, which means that the profits in Eq. (5) become

$$(\forall p \leq p^m) \pi_i = \frac{\alpha_i(p - mc) \prod_{j \neq i} (1 - \alpha_j F_j(p))}{1 - \prod_{j=1}^n (1 - \alpha_j)}. \tag{B1}$$

The denominator accounts for the consumers who fail to receive a price offer and then search again. The second effect is that the maximum price p^m is now the minimum of the choke price \bar{p} and the maximum that any consumer is willing to pay rather than search again, r . Moreover, since I assumed that consumers who fail to obtain the good choose to search, it follows that p^m equals r , since a consumer who fails to obtain the good and chooses to search has a nonnegative expected utility equalling $\bar{p} - r$. It is easy to see that r is just the expected price Ep plus the expected total search cost. This is because a consumer facing price r can continue searching at cost σ until obtaining a price, and expect to obtain the expected price, yielding:

$$r = Ep + \frac{\sigma}{1 - \prod_{j=1}^n (1 - \alpha_j)}.$$

That is, the reservation price is just the expected price plus the search cost times the expected number of searches until the good is obtained.

Finally, the price distributions computed in Lemma 2 are correct for this case. This arises because the profit function represented by (B1) is proportional to the profit function in (5), recalling that the α_i are given.*

* A large number of small steps are subsumed in this observation. First, facts (1)-(4) are correct, provided one defines p^m as the minimum of the choke price and the price which induces search. I have substituted $R(p) = p - mc$. This leads to (B1). From (B1), lemmas 1 and 2 then follow without alteration, with the adjustment for the proportionality of the profit functions, using the denominator in (B1).

However, an additional equation defining p^m must be added. First note that the expected price is equal to marginal cost plus the sum of the firms' expected profits per consumer. This leads to

$$p^m = r = mc + \frac{\sigma}{1 - \prod_{j=1}^n (1 - \alpha_j)} + \frac{\sum_{k=1}^n \alpha_k (p^m - mc) \prod_{j \neq k} (1 - \alpha_j)}{1 - \prod_{j=1}^n (1 - \alpha_j)},$$

or

$$p^m - mc = \frac{\sigma}{1 - [(1 - \sum_{k=2}^n \alpha_k) \prod_{j=2}^n (1 - \alpha_j)]}. \quad (\text{B2})$$

If the value of p^m given in (B2) exceeds \bar{p} , then the no search case arises. As in the no search case, profits per consumer contacted are invariant to the availability rate of the largest firm.

I now turn to endogenizing the availability rates. I assume that the consumers observe the values of the α_i 's, for otherwise equation (B2) would hold for the expectations of consumers, and I would have to distinguish between the consumer's expectations and the actual choices of the firms, which are not equal when a firm considers deviation.⁹ Adjusting (6) for consumers who fail to receive price offers (i.e., dividing by the denominator in (B1)), and substituting (B2), firm i 's *ex ante* expected profits are

$$\Pi_i = \frac{\alpha_i \sigma \prod_{j=2}^n (1 - \alpha_j)}{1 - [(1 - \sum_{k=2}^n \alpha_k) \prod_{j=2}^n (1 - \alpha_j)]} - c(\alpha_i).$$

An argument similar to the proof of Theorem 3 demonstrates the following.¹⁰

THEOREM 3'. *In a pure strategy equilibrium with search, $\alpha_1 > \alpha_2 = \alpha_3 = \dots = \alpha_n$.*

⁹ This is not an unreasonable assumption in the mall interpretation, for a consumer may know a firm's national market presence via national advertising, but not whether the firm is locally represented.

¹⁰ There is a possibility of a "regime shift," that is, by increasing the choice of α_i , a firm might induce consumers to search when they would not otherwise. This creates a discrete increase in demand, because consumers that receive no offers and hence do not purchase when searching is too expensive now choose to search and purchase. The conclusion of the theorem is not disturbed by this possibility, for the result follows from first-order conditions and holds in either regime. Regime shifts would not be possible if consumers can not observe the values of α_i , but infer them from the equilibrium values. But this leads to the possibility that a searching consumer might observe a price which is not possible given his expectation of the equilibrium, and then the consumers' off equilibrium conjectures will influence the equilibrium. I leave this interesting case for future research.

Unfortunately, the sort of analysis that the no search case yields with regard to scale economies does not seem possible in the case where consumers failing to obtain a price choose to search. However, in either case, equilibria involve firms choosing distinct values of α_i .

REFERENCES

1. K. BURDETT AND K. L. JUDD, Equilibrium price dispersion, *Econometrica* **51** (1983), 955–969.
2. G. R. BUTTERS, Equilibrium distributions of sales and advertising prices, *Rev. Econ. Stud.* **44** (1977), 465–491.
3. J. A. CARLSON AND R. P. MCAFEE, Discrete equilibrium price dispersion, *J. Pol. Econ.* **91** (1983), 480–493.
4. P. CRAMTON AND T. PALFREY, Cartel enforcement with uncertainty about costs, *Int. Econ. Rev.* **31** (1990), 17–48.
5. R. P. MCAFEE AND M. WILLIAMS, The department of justice merger guidelines: A critique and a proposed improvement, *Pepperdine Law Rev.* **16** (1989), 1069–1081.
6. R. P. MCAFEE AND J. MC MILLAN, Bidding rings, *Amer. Econ. Rev.* **82** (1992), 579–599.
7. J. ROBERT AND D. O. STAHL, Informative price advertising in a sequential search model, *Econometrica* (1992), in press.
8. H. R. VARIAN, A model of sales, *Amer. Econ. Rev.* **70** (1980), 651–658.