Production Capacity for Durable Goods

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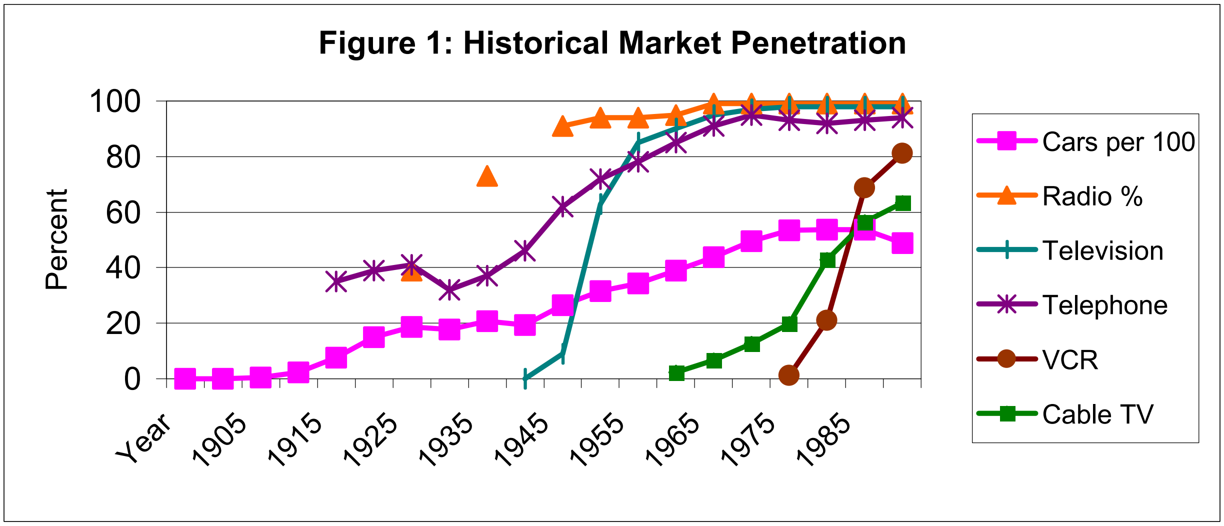
Compaq introduced the ipaq 3600 series handheld computer in mid-2000. Priced at $500, the device was rated as a best buy by C/Net, PC World and other industry sources. One year later, these devices are still in short supply, with resellers asking – and getting – $800.[1]

The ipaq 3600 series stands out because we have become accustomed to rapid provision of high technology goods, with the products reaching market saturation in a very short period of time, a matter of a few months or years. But how much production capacity is desirable? Is it efficient for firms to install sufficient manufacturing capacity to saturate the market in a year or less? Should we expect the Compaq story to be normal, with extended shortages of popular devices, or is it reasonable to expect market saturation to occur rapidly? How much capacity should a durable good seller possess?

Figure 1 presents data on the rate of U.S. penetration for six major consumer durables over this century. For all of these items, full market penetration took place over a decade or more. The producers of these items experienced a "soft landing," in the sense that sales did not plummet, but instead converged to replacement level. In contrast, full market penetration of the Citizen's Band radio took place over approximately five years, leading to a crash in CB radio sales in 1977. Unlike the case of CB radios, Figure 2 shows that sales of color televisions in the U.S never fell, even during recessions – the growth rate was always positive. Should we expect durable goods to have a soft landing, as occurred with televisions, when market saturation is reached, so that the replacement market is sufficient to sustain industry sales, or is it more likely that the sellers will experience a collapse along the lines of the "CB radio craze?" How long

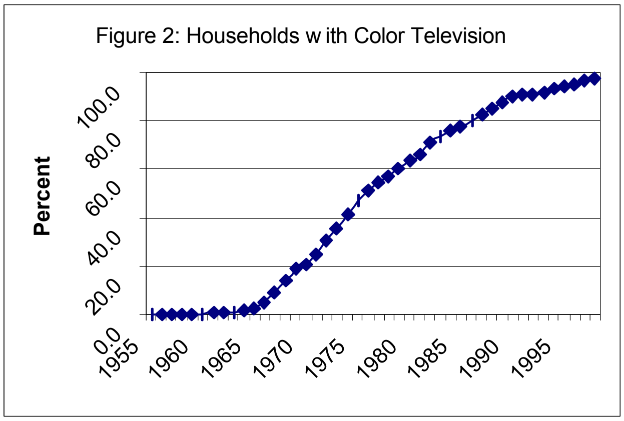
[1] There are actually two devices – the 3650 and the 3630. The distinction between these devices is the retail outlet – the 3630 is sold in "brick and mortar" stores while the 3650 is intended for sale over the internet. The initial plan, apparently, was to give different numbers to the devices so that customers who purchased the 3630 did not feel ripped off when they found out the 3650 was sold for less. However, the prices have actually inverted, with average internet prices for the 3650 being considerably higher than the physical store prices. Indeed, some sales of 3630 labeled devices are now transpiring on the internet.

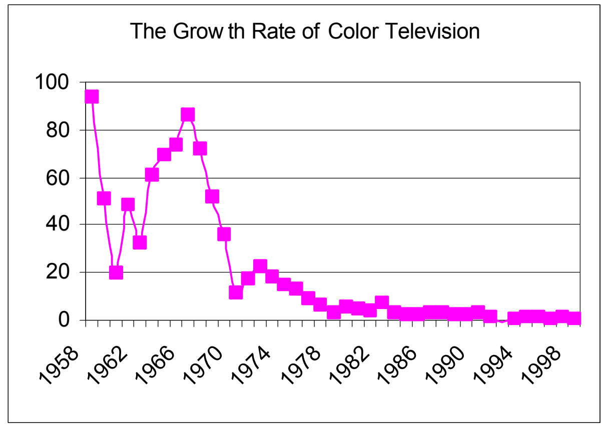


should it take for manufacturers of durable goods to satiate the marketplace?

This paper introduces a simple model to address such questions. The basic theory concerns a monopolist with no threat of competition facing identical consumers, and abstracts from the learning curve, substitute products and other features of durable goods manufacturing and sales, many of which work to slow market penetration relative to the rate predicted by the theory. The main conclusion of the theory is that, for common interest rates and other parameters, we should expect full market penetration to require ten to twenty years or more. Moreover, when the goods are imperfectly durable, full penetration can require fifty years or so. The theory suggests that penetration requires a long time relative to the actual rate of penetration of cellular phones, VCRs, camcorders, palm computing devices and other imperfectly-durable consumer durables. In such cases, the crash is small, while fast penetration necessarily requires that a significant crash occurs around the time of full market penetration, when the market switches from new sales to replacement sales.

The basic theory has the feature that market penetration is efficient, for the intuitive reason that the monopolist is capturing all the of the value of production, and thus desires to maximize that value and hence chooses an efficient capacity. Thus, the long times to market





penetration are a feature of efficiency as well as monopoly. The time to market penetration is decreasing in the durability of the good, and tends to be U-shaped in the interest rate. For low interest rates, there is little gain from fast penetration, because everyone is patient, and thus it pays to use capacity over longer times to satiate the market. For very high interest rates, the profitability of the market is reduced, and the firm slows market penetration in response, converging to an infinite time to market penetration for a finite interest rate that makes the production unprofitable.

Does competition speed up market penetration? We will show that in one sense, the answer is yes – the more firms there are, the faster is the market penetration. However, this increase in speed will not be adequate to overturn the conclusions of the basic theory. The basic theory considered a seller who did not undercut itself over time, even when the market reached saturation. With competition, such a path becomes implausible, and prices will tend to converge to marginal costs over time, a feature of the theory known as the Coase conjecture. It turns out that while competition accelerates market penetration, penetration converges to the basic theory solution as the number of firms goes to infinity. This is quite sensible: a monopolist that can capture all the value of its sales produces efficiently, as does the perfectly competitive industry; an imperfectly competitive industry is slower to saturate the market as a means of propping up the price. The case of monopoly divides into two types – the efficient monopolist and the monopolist who competes, imperfectly, with future incarnations of itself. The former is efficient, the latter the slowest to market of all.

Most relevant economic theory has been focused on Ronald Coase's wonderful 1972 conjecture that a monopolist of a durable good will have an incentive to cut the price, and when the monopolist can cut the price sufficiently rapidly, the monopolist will price near marginal costs. For example, Gul, Sonnenchein and Wilson (1986) demonstrate that the Coase conjecture is a feature of stationary equililbria. Kahn (1986) demonstrates that increasing marginal costs insure that even the continuous time limits of discrete time games have positive profits, although these profits are lower than those which would arise on the commitment path. By positing a fixed, albeit endogenous, initial capacity, we sidestep the Coase conjecture, because the seller *cannot* sell the large quantities required by the Coase path.

In one sense the Compaq ipaq story is unusual because Compaq did not price the 3600 series to capture the high prices created by the shortage. Consequently, an important part of the analysis of pricing concerns the optimal price path. We are used to the rapid decline in prices of consumer electronics. Prices may start high, but rapidly fall to a small fraction of their initial levels as mass production and competition take hold.[2]

A Basic Model of Monopoly

Consider the introduction of a new durable product by a monopolist. The product's durability, δ, is the rate at which the product fails; this is modeled for convenience as an exponential, so that a product sold at time t is still operating at time s with probability $e^{-\delta(s-t)}$. Let

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[2] It appears Compaq misjudged the popularity of its device, or perhaps was unable to obtain sufficient displays to meet the market projection, because it introduced a black and white version of the device after the initial introduction.

r be the rate at which future profits are discounted, so that profits received by the firm at time t have a present value of e^{-rt}.

Suppose that the flow value of the good to any customer is equal to the continuous consumption value of a competitively supplied non-durable good. This would be precisely the case if the durable good replaced a flow of a non-durable, such as an energy saving device. The expected present value of the cost of obtaining continuous use of the good, if the price is p, is the amount v satisfying

$$v = p + \int_0^{\infty} d e^{-dt} e^{-rt} v dt = p + \frac{d}{r+d} v,$$

as the probability density function of failure of the first purchase occurring at t is $\delta e^{-\delta t}$, at which point a new unit is purchased, incurring the discounted cost ve^{-rr}. This solves for a present cost:

$$v = \frac{r+d}{r} p.$$

The value v is determined by the pricing of an equivalent non-durable; let a constant infinite stream of a competitively supplied non-durable, such as labor, represent the numeraire good. If the non-durable is competitively supplied, the present value of cost of purchasing the non-durable will be $v = mc/r$, where mc is the marginal cost of the non-durable. Thus, setting this mc to \$1 without loss of generality, the maximum willingness to pay for the good is $\frac{1}{r+d}$. The flow value of benefits from the good is \$1.

Set marginal production costs to zero. If marginal costs are constant, then there is no loss of generality setting them to zero, because prices can be measured net of marginal costs.

The state variable of the system will be the proportion of the population that currently has the good, a variable denoted y. This variable has the equation of motion

(1) $\quad y'(t) = x - dy,$

where x is the production of the firm. This equation arises because the increase in the number of consumers holding the good is the flow of production x minus the consumers whose good depreciates, the number δy. Let c/r be the cost of the physical manufacturing facilities per unit of output; a production plant that produces x units per day costs cx/r to build.[3] The value c is the flow cost of the manufacturing facility if it were completely financed with debt. Moreover, for small δ, c is approximately equal to the so-called payback period – the number of periods prior at which the total dollars received equals the expenditure on the plant. The firm is assumed to choose x initially and not augment it later; this situation is justified if there are fixed costs associated with adding capacity.

If $(r+\delta)c \geq 1$, then it is not profitable to construct any manufacturing facility, because the present value of the proceeds is less than the cost of the facility. The present value of the proceeds from a plant of size x is, for sufficiently small x that the plant is fully utilized forever:

$$\frac{x}{r+d}\int_0^\infty e^{-rt}\,dt - \frac{cx}{r} = \frac{x}{r(r+d)} - \frac{cx}{r}.$$

Consequently, the interesting case is when

(2) $(r+\delta)c < 1$,

so that production is profitable, and this assumption is maintained for the remainder of the paper.

Given the nature of demand, the firm will buy all of its productive capacity initially at time 0 and produce at full capacity until the market is saturated, a time denoted T. After that time, the firm will only sell to the replacement market, which is the fraction δ of the total population. The equation of motion can be solved for T given the initial condition $y(0)=0$ and the capacity x by integrating and solving for $y(T)=1$. This calculation yields:

[3] The cost of the facility takes the form c/r to denominate it in terms of the labor or non-durable units used for the

Footnote continued on next page…

(3) $$T = \frac{Log(x/(x-d))}{d}$$

or

(4) $$\frac{d}{x} = 1 - e^{-dT}.$$

The firm's profits arise from sales of x for T periods, plus sales of δ for the remainder of time, minus the cost of capacity. (Recall that marginal costs of production were already subtracted from the payments of customers, so that the only remaining costs are capacity costs.) Thus the firm's profits are:

(5) $$p = \frac{1}{r+d}\int_0^T e^{-rt} x dt + \frac{1}{r+d}\int_T^\infty e^{-rt} d dt - \frac{cx}{r} = \frac{x}{r(r+d)}(1-e^{-rT}) + \frac{d}{r(r+d)} e^{-rT} - \frac{cx}{r}.$$

In the expression for profits, T and x are related by equation (4). However, it will prove practical to think of the firm as choosing T, with equation (4) determining x. In this case, profits can be expressed as

(6) $$rp = \frac{r}{r+d}\frac{d}{r}\left[\frac{1-e^{-rT} - (r+d)c}{1-e^{-dT}} + e^{-rT}\right] = \frac{d}{r+d}\left[\frac{1-e^{-(r+d)T} - (r+d)c}{1-e^{-dT}}\right].$$

This expression is maximized when the first order condition yields:

(7) $$c = \frac{d - (r+d)e^{-rT} + re^{-(r+d)T}}{d(r+d)}$$

The right hand side of this expression ranges from 0 to $1/(r+\delta)$ as T ranges from 0 to ∞ and is increasing in T. Thus, there is a unique solution T^* to (7) if and only if (2) holds. Moreover, T^* increases in the cost of capacity – when capacity is more expensive, it takes longer for a monopolist to saturate the market. That the saturation time increases as capacity becomes

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revenues; $\$1/r$ purchases a fixed stream of non-durables, the numeraire.

more expensive is a feature of any reasonable model.

As depreciation rises, the optimal saturation time T increases, a fact that is demonstrated in Lemma A1 of the appendix. Consequently, the optimal value of T is minimized when δ=0. The case when δ=0 is the case of a perfectly durable good, and in this instance, T^* satisfies:

(8) $$rc = 1 - e^{-rT^*}(1 + rT^*).$$

Equation (8) can be used to prove two interesting heuristics for the saturation of a market.

Theorem 1: The optimal time for market saturation, T^*, satisfies:

(9) $$T^* \geq \frac{2.6}{r}(rc)^{0.718}, \text{ and}$$

(10) $T^* \geq 3.35\ c$.

Proof: It is simplest to work with (8), which gives a lower bound to T^*. Define $y=rT$ and $a=1/(e\text{-}2)\approx 1.392$. Rewrite (8) to obtain

(11) $$\frac{rc}{(rT)^a} = \frac{1 - e^{-x}(1+x)}{x^a} \equiv h(x) \leq 1 - \frac{2}{e} \approx 0.264241.$$

The inequality in (11) arises from maximization of h over x, a maximization that results in x=1. (In fact, a was chosen so that the maximization of h results in x=1. Because a<2, h(0)=0.) From (11),

$$T \geq \frac{(rc)^{1/a}}{r(1 - 2/e)^{1/a}} = \left(\frac{e}{e-2}\right)^{e-2} \frac{(rc)^{e-2}}{r} \geq 2.601 \frac{(rc)^{.718}}{r}.$$

The derivation of (10) is accomplished as follows. Rewrite (8) to give

$$\frac{rTc}{T} = 1 - e^{-rT}(1 + rT).$$

Consequently,

(12) $$\frac{c}{T} = \frac{1 - e^{-rT}(1 + rT)}{rT} \leq 0.298427 \leq \frac{1}{3.3508}.$$

The inequality (12) readily proves (10). *Q.E.D.*

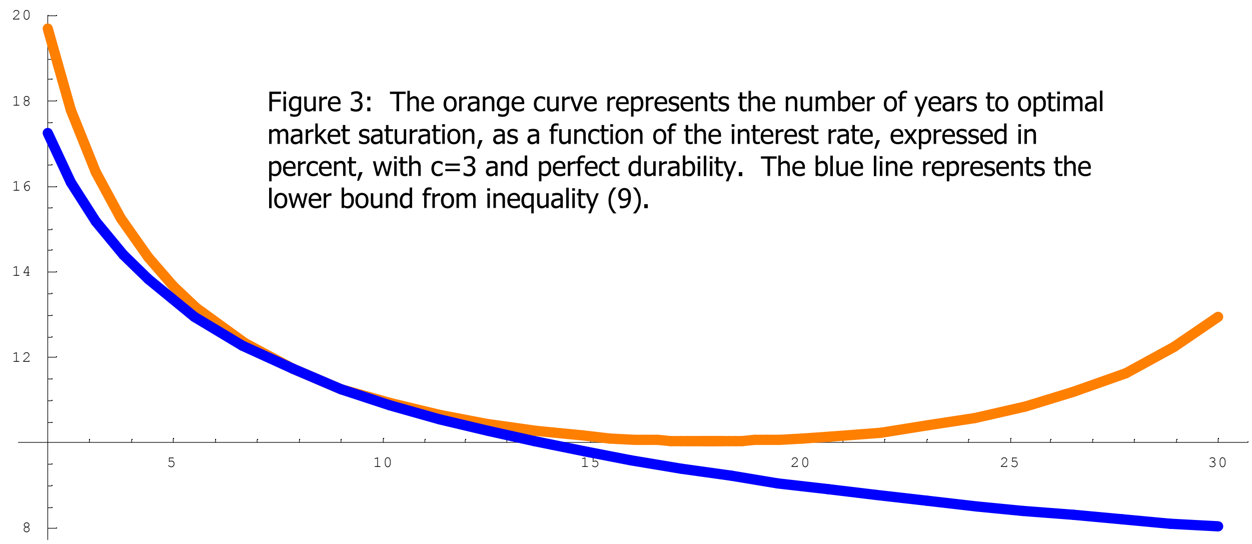

Figure 3: The orange curve represents the number of years to optimal market saturation, as a function of the interest rate, expressed in percent, with c=3 and perfect durability. The blue line represents the lower bound from inequality (9).

Theorem 1 provides two heuristics for the optimal time to reach market saturation for the case of a perfectly durable. Inequality (10) is the most straightforward, placing a lower bound on the time to saturation for any interest rate. The interesting fact about this lower bound is the absolute magnitude – the lower bound is quite large. Recall that *c* is the cost of a manufacturing facility per dollar of profit generated. Thus, a value of c=3 corresponds to a pay-back period of 3 years.[4] In this case, the time to market saturation is at least a decade! In general, the time to market saturation is at least $3\,{}^{1}\!/_{3}$ times the undiscounted pay-back length of time. For moderate interest rates, the optimal length of time can be substantially longer, with two decades arising in the case of very low interest rates. These values are illustrated in Figure 3.

An interesting aspect of the problem is visible in Figure 3. For very low interest rates, the firm is patient, so that the value of fast market penetration is small. Thus, as the interest rate falls toward zero, the optimal penetration time converges to infinity. This is not a consequence of the increase in manufacturing costs, because the willingness to pay by consumers is rising at the same rate. Moreover, as the interest rate approaches $1/c$, the profitability of the project is

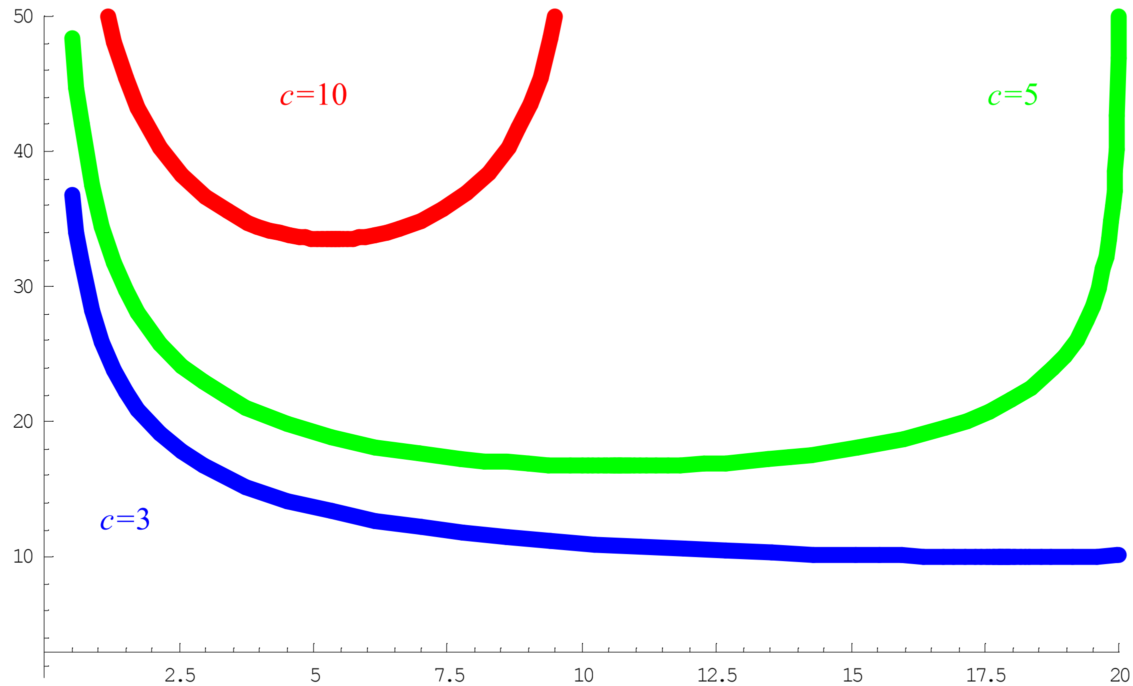

Figure 4: The number of years until market saturation for various costs of manufacturing as a function of the interest rate in percent, with δ=0.

declining, and the terminal time for saturation diverges.

Figure 4 explores the U-shaped optimal penetration time and illustrates the effect of various costs, for δ=0, as a function of the interest rate. For low interest rates, the optimal penetration time is high, because the seller is patient and doesn't care that it takes a long time to sell to the market. As the interest rate gets high, the entire project is becoming less and less worthwhile, thus increasing the time to penetrate the market, indeed sending it to infinity as $r \to 1/c$.

Depreciation of the good reduces the profitability of sales by reducing the buyer's willingness to pay. However, depreciation also increases the ultimate demand, increasing the long-term profitability of a larger capacity. On balance, the former effect dominates, and the

[4] This is standard, if weird, usage – the payback period is the length of time until the number of dollars – undiscounted – received in return equals the investment. If δ=0, this value equals c.

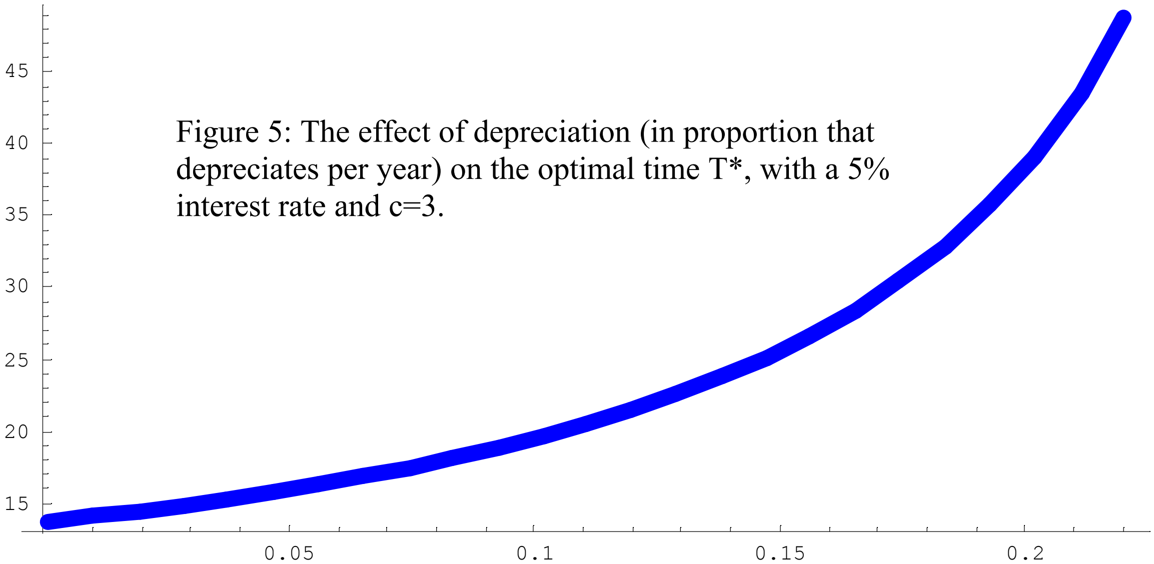

Figure 5: The effect of depreciation (in proportion that depreciates per year) on the optimal time T*, with a 5% interest rate and c=3.

effect of depreciation is to increase the optimal time before full penetration is reached. Figure 5 illustrates this effect when $c = 3$ and $r = 0.05$ – a five percent interest rate. The terminal time rises slowly for modest depreciation rates, but as the depreciation rate rises to 25% – so that the good has an expected life of four years – the terminal time rises to fifty years.

The implication of these numbers are very surprising. A monopoly seller of a fairly durable good, one that lasts four years, facing no Coase conjecture problems with price erosion, and $c = 3$, will take fifty years to penetrate the market. Part of the reason is that the pay-back period with a large depreciation rate, which reduces the willingness to pay, increases the payback period substantially, to eighteen years.

Capacity has an inverse relationship to market saturation, given by (4). Thus, capacity is upside-u-shaped in the interest rate, and decreases in the cost of capacity. For sensible parameters, it appears capacity increases in the depreciation rate of the good, in spite of the fact that the time to saturation also increases.

The main conclusion of the basic theory is that the optimal time for market saturation is quite long – substantially longer than the time to saturation usually observed in real-world durable goods. There could be several reasons for this. First, competition will increase the speed of market penetration by creating a race – the firm that brings more to market captures the lion's share of the rents. Second, even a monopoly will introduce good substantially faster if slow penetration is likely to attract entry – firms may increase market penetration to reduce the incentive of other firms to engineer competing solutions. In the subsequent section, we explore the former hypothesis – what is the effect of competition on the speed of market penetration?

The Basic Model with Competition

Suppose there are *n* firms with the technology to provide the good to the market. We maintain the prevailing assumptions on the technology and demand, with the provision that there is no longer monopoly. Each firm will choose a capacity x_i, and will choose a quantity $q_i(t)$ to provide to the market, subject to the restriction that $q_i(t) \leq x_i$. We will only consider equilibria in which all the firms engage in full production, given their capacities; this assumption is warranted by the analysis of Kreps and Scheinkman (1983). Firms are assumed to choose their initial capacities and not augment capacity later. This restriction is made for computability, but can be justified by positing some fixed costs associated with capital investment or a "putty-clay" technology that cannot be augmented.

In contrast to the monopoly analysis, if $d < \sum_{i=1}^{n} x_i$, then prices must be driven to zero (marginal cost) once the market is saturated, which occurs at time *T* given by (3), where *x* is replaced with the sum of the x_i. Let $p(t)$ represent the price charged to customers, so that $p(t)=0$ for $t \geq T$. The prices are determined by the fact that consumers must be indifferent to waiting to purchase, which produces utility $e^{-rT}\left(\frac{1}{r+d}\right)$, because the present value of the good to the customer is $1/(r+\delta)$. Thus, the prices are determined by the equation:

(13) $$\frac{e^{-rT}}{r+d} = \int_t^T d e^{-d(s-t)} \left(\frac{e^{-rt} - e^{-rs}}{r} + \frac{e^{-rT}}{r+d} \right) ds + e^{-d(T-t)} \left(\frac{e^{-rt} - e^{-rT}}{r} + \frac{e^{-rT}}{r+d} \right) - e^{-rt} p(t).$$

The left hand side of (13) is the value of purchasing at *T*. This value arises by waiting until *T* to purchase for a price of zero. Alternatively, if the agent buys at *t*, the good survives until *T* with probability $e^{-\delta(T-t)}$, and in this event, because of the exponential failure, the future value to the customer is the same as if they bought at time *T*, plus the customer has the flow value of the good from *t* to *T*. If the good fails in the interval $[t,T]$, at time *s*, which has density $\delta e^{-\delta(s-t)}$, the customer gets the flow value to date, plus the value of waiting to purchase at *T*. Equation (13) reduces to

(14) $$p(t) = \frac{1}{r+d}(1 - e^{-(r+d)(T-t)}).$$

Equation (14) provides the prices that make a customer indifferent between purchasing during the phase of production where production is constrained by capacity and waiting until production is unconstrained and prices fall to marginal cost, which was set to zero.

The profits earned by firm *i*, then, are

(15) $$p_i = \int_0^T e^{-rt} p(t) x_i dt - \frac{cx_i}{r} = \frac{x_i}{r+d} \int_0^T e^{-rt} - e^{dt-(r+d)T} dt - \frac{cx_i}{r}$$

$$= \frac{x_i}{r} \left(\frac{1-e^{-rT}}{r+d} - \frac{re^{-rT}(1-e^{-dT})}{d(r+d)} - c \right) = \frac{x_i}{r} \left(\frac{1-e^{-rT}}{r+d} - \frac{re^{-rT}}{(r+d)\sum_{j=1}^{n} x_j} - c \right)$$

where $X = \sum_{j=1}^{n} x_j$. Differentiating (4),

(16) $$\frac{dT}{dX} = -\frac{e^{dT}}{X^2} = -\frac{e^{dT}(1-e^{-dT})^2}{d^2}.$$

Thus,

(17) $$\frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_i}\frac{x_i}{r}\left(\frac{1-e^{-rT}}{r+d} - \frac{re^{-rT}}{(r+d)X} - c\right)$$

$$= \frac{1}{r}\left(\frac{1-e^{-rT}}{r+d} - \frac{re^{-rT}}{(r+d)X} - c\right) + \frac{x_i}{r(r+d)}\left(\frac{re^{-rT}}{X^2} + r\left(1+\frac{r}{X}\right)e^{-rT}\frac{dT}{dX}\right)$$

$$= \frac{1}{r(r+d)}\left[1-e^{-rT} - \frac{re^{-rT}}{X} - c(r+d) + \frac{x_i e^{-rT}}{X^2}\left(r - r\left(1+\frac{r}{X}\right)e^{dT}\right)\right]$$

$$= \frac{1}{r(r+d)}\left[1-e^{-rT}\left(1+\frac{r}{X}\right) - c(r+d) + \frac{rx_i e^{-rT}}{X^2}\left(1-\left(1+\frac{r}{X}\right)e^{dT}\right)\right]$$

Note that marginal profits are decreasing in the market share x_i/X, which implies that the only candidates for equilibria involving finite saturation time are symmetric – firms with larger market shares have strictly lower marginal profits. Thus, an equilibrium (with $X>\delta$) must satisfy:

(18) $$0 = 1-e^{-rT}\left(1+\frac{r}{X}\right) - c(r+d) + \frac{re^{-rT}}{nX}\left(1-\left(1+\frac{r}{X}\right)e^{dT}\right).$$

Equation (18) reduces to:

(19) $$1-c(r+d) = e^{-rT}\left[\left(1+\frac{r(1-e^{-dT})}{d}\right)\left(\frac{n-1}{n} + e^{dT}\frac{r(1-e^{-dT})}{nd}\right) + \frac{1}{n}\right].$$

In the limit as n diverges, this expression replicates the solution to the basic monopoly problem. This sounds strange, however, it arises because consumers are identical, which makes the monopolist a perfectly price-discriminating monopolist. As is well-known, a perfectly price-discriminating monopolist is efficient about production, and consequently the monopoly solution and the zero-profits competitive solution coincide; the difference is in the prices charged in the two cases.

Theorem 2: If $r>\delta$, the unique equilibrium is an interior solution to (19) with $T<\infty$. If $r\le\delta$ and $n\le 2$, any equilibrium involves $X=\delta$ and $T=\infty$. If $r<\delta$ and $n>2$, either kind of equilibrium is possible, depending on c. Finally, if $r=\delta$, and $n>2$, the equilibrium is an interior solution to (19) with $T<\infty$.

Proof: First note that X is decreasing in T, and moreover that profits are continuous in the limit as $X\to\delta$. It is readily established that profits are increasing in x_i for $X<\delta$, so the only candidates for equilibria involve $T\in[0,\infty]$. Thus we can work with (19) to characterize firm's incentives globally. (What makes this a bit tricky is that (19) is expressed in terms of T, and T is decreasing in x_i.)

The right hand side of (19) is 1 at $T=0$. The slope of the RHS is given by

$$\frac{\partial RHS}{\partial T}=\frac{r(r+d)e^{-rT}(1-e^{-dT})}{d^2 n}((d-r)e^{dT}+r-d(n-1)).$$

Thus, if $r>\delta$, the RHS converges to 0 as $T\to\infty$, and there will be an equilibrium given that the maintained hypothesis (2) is satisfied. Moreover, note that any extreme point to RHS is a maximum – the second derivative, evaluated where the first derivative is zero, is proportional to δ-r, which is negative by hypothesis. The only valid candidates for equilibria involve a decreasing RHS – this corresponds to the second order condition of the maximization problem of the firm.

If $r<\delta$, the RHS diverges as $T\to\infty$. Moreover, every extreme point is a minimum. Thus, the RHS is always at least unity if it is nondecreasing at $T=0$, which occurs if $n\le 2$. In this case, profits are increasing in T, which is equivalent to profits decreasing in x_i when $X>\delta$. Hence all equilibria involve $X=\delta$ and $T=\infty$. If $r<\delta$ and $n>2$, there will be locally optimal (RHS decreasing) solutions to (19) when c is large enough, and won't when c is sufficiently small.

The final case arises when $r=\delta$. If $n=1$, the RHS is increasing and the only solution involves $X=\delta$. If $n=2$, the RHS is constant at 1 and thus any equilibrium involves $X=\delta$. Finally, if $n>2$, the RHS falls monotonically to zero and thus is equivalent to the case when $r>\delta$. *Q.E.D.*

Intuitively, when δ is large, it doesn't pay for any firm to increase capacity beyond the point where prices start to fall, because the replacement market is sufficiently large that a larger share of a market with lower prices doesn't pay. In contrast, when δ is small, firms will prefer to sell more even at the cost of lower prices terminating at zero. The more firms there are, the

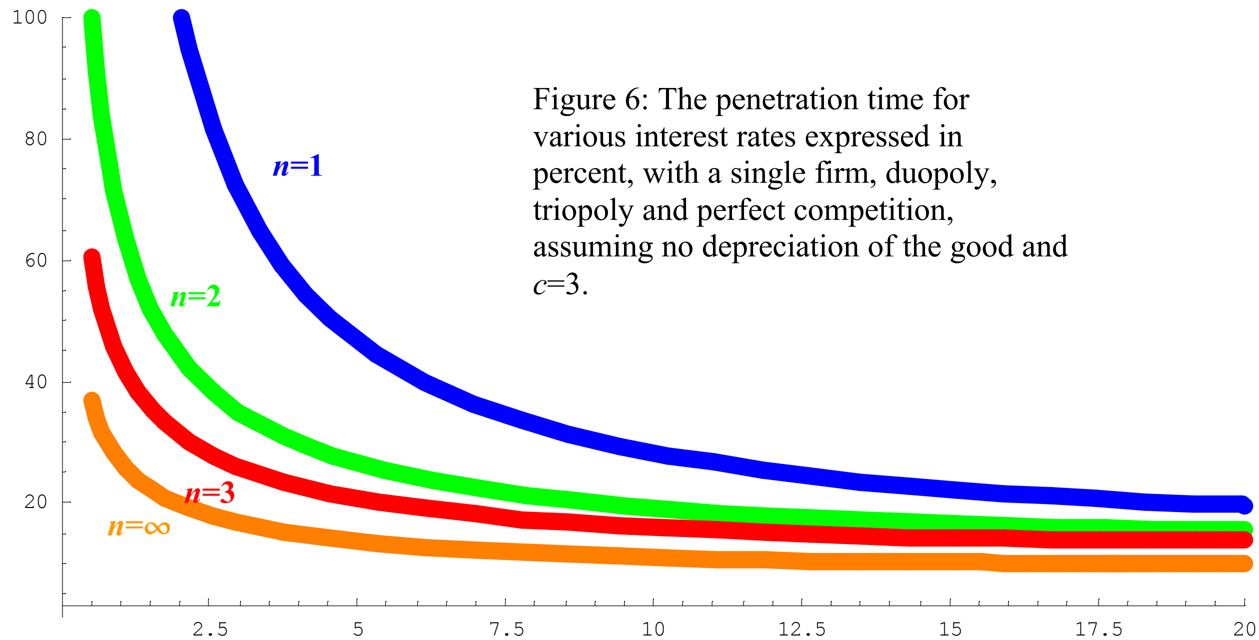

Figure 6: The penetration time for various interest rates expressed in percent, with a single firm, duopoly, triopoly and perfect competition, assuming no depreciation of the good and c=3.

stronger the incentive to increase capacity beyond δ even if this leads to declining prices.

When $T<\infty$, there are no asymmetric equilibria. In the case $\delta=X$, however, there may be asymmetric equilibria – the requirement that a firm doesn't find it profitable to increase x_i is an inequality in market share that holds strictly around symmetry.

The case n=1 can be interpreted as a Coasian path. (See Gul, Sonnenschein and Wilson(1986) and Coase(1972).) Along this path, the monopolist cuts his price because buyers won't buy unless he cuts his price, eventually driving prices to zero. Profits are positive because the firm cannot satisfy all the demand instantaneously, and profits exceed competitive levels because the monopolist chooses to restrict capacity so as to influence beliefs by customers. Nevertheless, pessimistic conjectures imply decreasing prices.

$$\frac{\partial RHS}{\partial n} = -\frac{e^{-rT}(1-e^{-dT})^2 r(r+d)}{d^2 n^2} < 0.$$

Thus, under the circumstances that T is finite, T is decreasing in n – more competition decreases the time before market saturation. For example, if δ=0, the expression defining T reduces to

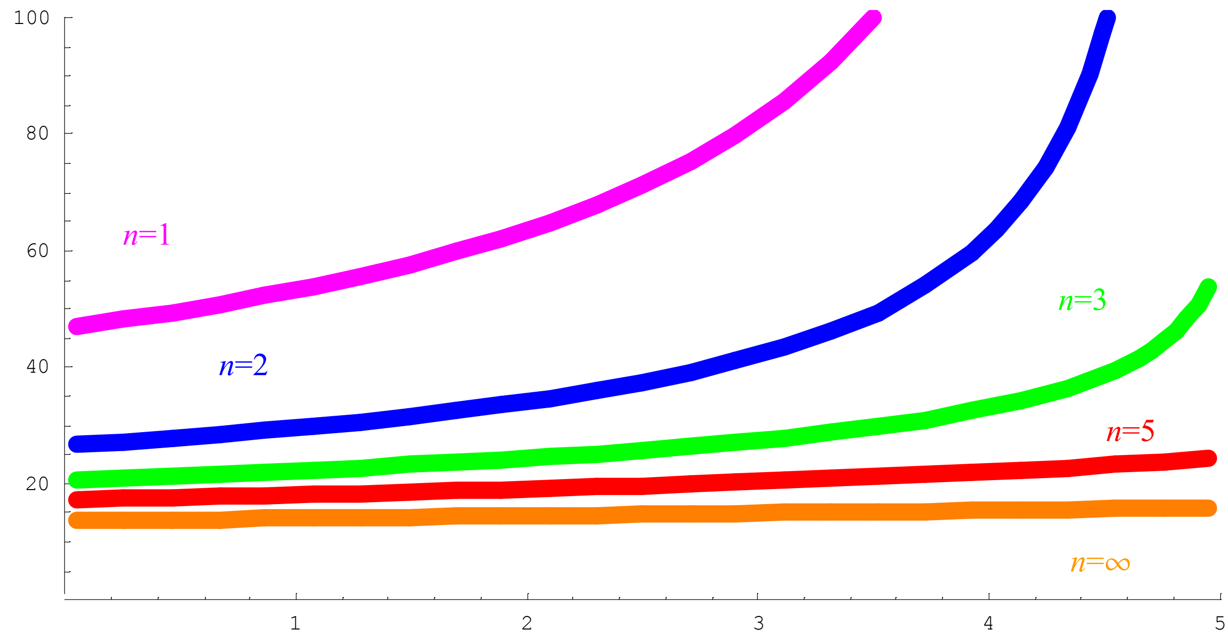

Figure 7: The saturation time as a function of δ (in percent), with a 5% interest rate and c=3.

$$1 - rc = e^{-rT}\left(1 + rT + \frac{(rT)^2}{n}\right).$$

A plot of the solution to this equation, for various values of n is provided in Figure 6. It is striking how long full market penetration takes. In contrast to the basic monopoly model with monopoly prices (which encourage investment in capacity due to high prices), competition *increases* the time to full market saturation, equaling the monopoly only in the limiting case of perfect competition. On the Coasian path, the monopolist who succumbs to the temptation to cut prices winds up cutting capacity dramatically, as a means of committing to higher prices than arise with larger capacity. With no depreciation, a payback period of 3 years and a 5% interest rate, it takes fifty years for the market to reach saturation.

The value of T appears to increase in the failure rate δ, which is sensible given that the required capacity to saturate the market rises as δ rises. Figure 7 illustrates the saturation times with a 5% interest rate and c=3. The effect of an increase in δ is generally modest for five or more firms. However, with only a few firms, increasing δ increases the saturation time significantly.

Conclusion

This paper presents a theory of manufacturing capacity choice for a durable good. The remarkable conclusion is that efficient production may entail ten to fifty years before full market saturation is reached. The time to market saturation is increased as the good becomes less durable, and the size of the crash when saturation is reached falls as the durability decreases. The monopoly seller is efficient provided he doesn't ever undercut himself, a feature of some equilibria of the "gap" case, where demand exceeds marginal costs.

With competition, either on the Coase path for a monopolist, or with multiple producers, market penetration may arrive only in the limit as time diverges, with sellers producing only the amount that replaces a satiated market. This situation arises only if the depreciation rate of the good is larger than the interest rate, and may not arise when the number of competitors exceeds two, depending on the cost of capacity. In such a case, there is no crash, only a soft landing as the market is satiated, with a growth rate converging to zero.

Increases in the number of competitors speed product introduction, converging to the efficient level as the number of competitors goes to infinity. In addition, increases in the depreciation rate of the good also tend to increase the time to market saturation.

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Technical Appendix

Lemma A1: T^* is increasing in δ.

The value of T^* is determined by (7). It is readily established that the RHS of (7) is increasing in T, so it suffices to show that the right hand side of (7) is decreasing in δ Define a function γ by the right hand side of (7), that is,

$$g(d,T) = \frac{d-(r+d)e^{-rT} + re^{-(r+d)T}}{d(r+d)} = \frac{1}{r+d} - \frac{e^{-rT}}{d} + \frac{re^{-rT}}{d(r+d)}.$$

Then

$$\frac{\partial g}{\partial d} = \frac{-1}{(r+d)^2} + \frac{e^{-rT}}{d^2} - \frac{re^{-(r+d)T}}{d(r+d)}\left(\frac{1}{d} + \frac{1}{r+d} + T\right)$$

It is straightforward to show that $\partial\gamma/\partial\delta = 0$ at T=0; indeed, $\gamma(\delta,0)$=0. Thus, it suffices to show that $\partial\gamma/\partial\delta$ is decreasing in T to show that $\partial\gamma/\partial\delta \le 0$. But

$$\frac{\partial}{\partial T}\frac{\partial g}{\partial d} = -\frac{re^{-rT}}{d^2} + \frac{re^{-(r+d)T}}{d(r+d)}\left(-1 + (r+d)\left(\frac{1}{d} + \frac{1}{r+d} + T\right)\right)$$

$$= \frac{1}{d^2(r+d)}\left[-(r+d)re^{-rT} + re^{-(r+d)T}\left(-d + (r+d)\left(1 + \frac{d}{r+d} + dT\right)\right)\right].$$

$$= \frac{re^{-rT}}{d^2(r+d)}\left[-(r+d) + e^{-dT}\left((r+d)(1+dT)\right)\right]$$

$$= \frac{-(r+d)re^{-rT}}{d^2(r+d)}\left[1 - e^{-dT}(1+dT)\right] \le 0. \qquad \text{Q.E.D.}$$

Simplification of (13): Rewrite (13) to obtain

$$0 = \int_t^T de^{-d(s-t)}\left(\frac{e^{-rt} - e^{-rs}}{r}\right)ds + e^{-d(T-t)}\left(\frac{e^{-rt} - e^{-rT}}{r}\right) - e^{-rt}p(t)\text{, or}$$

$$rp(t) = \int_t^T de^{-d(s-t)}\left(1 - e^{-r(s-t)}\right)ds + e^{-d(T-t)}\left(1 - e^{-r(T-t)}\right)\text{, or,}$$

$$rp(t) = (1 - e^{-d(T-t)}) - \frac{d}{r+d}(1 - e^{-(r+d)(T-t)}) + e^{-d(T-t)}\left(1 - e^{-r(T-t)}\right)\text{, or,}$$

$$rp(t) = 1 - \frac{d}{r+d}(1 - e^{-(r+d)(T-t)}) - e^{-d(T-t)}e^{-r(T-t)} = \frac{r}{r+d}(1 - e^{-(r+d)(T-t)})$$

Simplification of (18): Rewrite (18) and substitute (4) to obtain:

$$1 - c(r+d) = e^{-rT}\left[\left(1 + \frac{r}{X}\right) - \frac{r}{nX}\left(1 - \left(1 + \frac{r}{X}\right)e^{dT}\right)\right]\text{, or,}$$

$$1 - c(r+d) = e^{-rT}\left[\left(1 + \frac{r(1 - e^{-dT})}{d}\right) - \frac{r(1 - e^{-dT})}{nd}\left(1 - \left(1 + \frac{r(1 - e^{-dT})}{d}\right)e^{dT}\right)\right]\text{, or,}$$

$$1 - c(r+d) = e^{-rT}\left[\left(1 + \frac{r(1 - e^{-dT})}{d}\right)\left(1 + e^{dT}\frac{r(1 - e^{-dT})}{nd}\right) - \frac{r(1 - e^{-dT})}{nd}\right]\text{, or,}$$

$$1 - c(r+d) = e^{-rT}\left[\left(1 + \frac{r(1 - e^{-dT})}{d}\right)\left(\frac{n-1}{n} + e^{dT}\frac{r(1 - e^{-dT})}{nd}\right) + \frac{1}{n}\right],$$

as desired.