

An Application of Complexity Theory to the Analysis of Internal Control Systems

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ABSTRACT

Complexity Theory provides a means of evaluating the computing time requirements and the amount of memory space necessary to solve problems. The method has previously been employed most often in Computer Science in analyzing computing time requirements for computer programs. While internal control systems are not computer programs as such, their structure is essentially of the same nature as computer programs. We demonstrate the appropriateness of applying Complexity Theory to issues of internal control analysis. More specifically, we argue that, as written, the 1977 Foreign Corrupt Practice Act may impose theoretically unacceptable costs of analysis on the accounting and auditing professions with respect to its internal control requirements.

THE Foreign Corrupt Practices Act of 1977, or FCPA, amends the Securities and Exchange Act of 1934 to require, among other things, sec. 102 (2) "Every issuer of a class of securities registered pursuant to section 12 of this title . . . shall (B) devise and maintain a system of internal accounting controls sufficient to provide reasonable assurance that (i) transactions are executed in accordance with management's general or specific authorization, (iii) ac-

cess to assets is permitted only in accordance with management's general or specific authorization" [5].

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The law specifies that "reasonable assurance" is required and not absolute assurance. It is unclear how reasonable assurance is to be established and thus what will conform with the FCPA. The Act assigns the determination of compliance to the professional accounting community, the Securities and Exchange Commission (SEC) and the courts — primarily the courts. This is perhaps not surprising as accounting practices were the primary domain of the accountants and auditors [8] and [9].

Harold Williams, Chairman of the SEC, [PMM 1981] has indicated his concerns associated with the FCPA have been overruled. If the SEC is made up of reasonable people, they will apply reasonable rules. We can expect that SEC action will be necessary to clarify a "vague" wording of the Act. Unfortunately, it remains to be seen if this will reduce the accounting profession's influence with respect to the FCPA. Douglas Carmichael notes that the SEC sounds as if it's benevolent and that the enforcers that's being offered are dependent upon the SEC. Those in charge will always worry."

Accountants and auditors are generally in agreement that the interpretation of the FCPA is extremely vague. As a result, they have their own interpretations of the FCPA, expressed in their pronouncements and review procedures [10]; AA, 1976 [1]; AA, 1980 [10]; AA, 1976 [1]; AA, 1980 [10]. It is unclear as to whether the courts will rule that developing and auditing internal controls will be sufficient to meet the requirements of the FCPA. The interpretation of business and the ac-

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The law specifies that "reasonable assurance" is required under the FCPA, not absolute assurance. It is not clear as to how reasonable assurance will be established and thus what will constitute compliance with the FCPA. The law effectively assigns the determination of the terms of compliance to the profession, lawyers, the Securities and Exchange Commission (SEC) and the courts — primarily the SEC and the courts. This is perhaps unfortunate as accounting practices were previously the primary domain of the accountants, and auditors [8] and [9].

Harold Williams, Chairman of the SEC, [PMM 1981] has indicated that the concerns associated with the impositions of the FCPA have been overstated, that the SEC is made up of reasonable men who will apply reasonable rules. Nevertheless, we can expect that SEC and court action will be necessary to clarify and interpret the "vague" wording of the FCPA. Admittedly, it remains to be seen as to whether this will reduce the accounting professions influence with respect to such matters. As Douglas Carmichael notes in [10], "It sounds as if it's benevolence of the enforcers that's being offered. Any institution depending upon the benevolence of those in charge will always have cause to worry."

Accountants and managers are generally in agreement that a strict interpretation of the FCPA is extremely burdensome. As a result, they have supplied their own interpretations of the requirements of the FCPA, expressed in their various pronouncements and review programs PMM, 1980 [10]; AA, 1976 [1]; AY, 1980 [2]. It is unclear as to whether the SEC and the courts will rule that developing practices in designing and auditing internal control systems will be sufficient to meet the requirements of the FCPA. The increasing regulation of business and the active judicial pro-

cess makes it feasible to anticipate that the courts will rule that the legislators intend "reasonable assurance" to mean that every possible avenue for material internal control failure be blocked in some way. Strict interpretations would involve high levels of "reasonable assurance" and small values for the materiality measures. Such interpretations are clearly in the set of possible court interpretations. When an internal control system does fail, the SEC and the courts, having the advantage of hindsight, may find it convenient and perhaps too easy to criticize the internal control system and those responsible for it for not having provided reasonable assurance. One of the primary points of this paper is that such internal control failures must be expected on occasion, because it is prohibitively expensive to prevent such failures.

When the government regulates industrial pollution or requires that safety features be built into products, businesses can generally calculate the cost of the regulation. The feasibility of this computation provides a basis for a cost/benefit analysis. Despite the difficulties of benefits measurements, the availability of cost data tends to contribute to the identification of feasible regulations. The regulation of accounting controls does not permit a straightforward cost computation of this nature. One can compute the cost of current auditing practices by averaging the costs of current audits. However, computing the costs of proposed auditing standards does not permit such an empirical consideration. If the technological methods were available and the FCPA had specified quantifiably attainable criteria against which to compare, a cost computation similar to that used to estimate the cost of particulate pollution reduction might be feasible. The FCPA is intentionally vague as to quantifiable criteria. "Reasonable-assurance" is an ill-defined concept and audit technology

is inadequate to any but a broad interpretation of "reasonable assurance." It has served well where the internal control issue was only one part of the larger audit, but its serviceability in the more focussed setting of the FCPA, is questionable. Due to current technological limitations and the vagueness of the FCPA another approach to cost estimation may be worth consideration.

This paper uses an approach to cost estimation appropriate to the FCPA's impact on the study of internal controls, i.e., when the type of problem has been defined, but the method of solution is permitted to vary. It is a cost estimation method often used by computer scientists. The application to internal control and FCPA issues is designed to acquaint the reader with the approach as well as to consider the implications of strict interpretations of the FCPA. We will prove that under strict interpretations of the FCPA the burden placed on management and the auditor in designing and auditing internal control systems is not cost beneficial. It is not cost beneficial because the problem is too complex to be solved in any reasonable amount of time. We recognize that by taking a strict interpretation position in this presentation, we set an upper bound on the difficulty of solution. No pragmatic solutions are so strict, as we have been reminded by academic and practitioner alike. Our point is exactly that and a bit more — no pragmatic solution can be that strict.

The method of proof employed in this paper is called Complexity Theory. Complexity Theory has been employed in Mathematical Logic and Computer Science where its purpose has been to evaluate the computing time requirements and the amount of memory space necessary to solve certain problems. It turns out that some very dissimilar problems have similar computing time requirements. In particu-

lar, there are known groups of problems, all of which have similar computing time requirements.

While internal control systems are not computer programs, the structure of internal control systems are such that complexity theory can be appropriately applied. Specifically we demonstrate that the accounting and auditing problem embodied in the FCPA is a member of a class of problems called "NP complete" problems. The only known algorithmic solution procedures for NP complete problems has a cost function which is exponentially related to the input size, at least for worst case problems. Moreover, given the extent of research to date involving NP complete problems, it seems unlikely that a nonexponential solution will be found. NP complete problems have the property that if one can be solved in polynomial time. For this reason, NP complete problems are considered intractable, that is, only the small size problems can be solved in a reasonable amount of computing time, i.e., less than exponential time, then they all can be solved in less than exponential time. If the FCPA of 1977 requires the accountant to solve an internal control problem that is NP complete and large, then the FCPA requirements are more than costly; they are practically impossible to satisfy.*

A computer program is an organized specification of behavior. Consequently, in cases where behavior may be modelled and analyzed, the tools of analyzing computer algorithms are appropriate. This provides a large body of mathematical results and tools which are appropriate and applicable even if, at present, rarely seen outside of Computer Science. Although Complexity

*The term "large" is not well defined. However, size can be established given a specific input and the assessment of the cost of solution established by complexity theory. Thus the issue of largeness will be resolved later in the paper.

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Finally, section 5 presents implications, some other method of analysis, and an of the application to the FCPA is an interesting example of of Complexity Theory to However, there are many a in Accounting for which Theory is relevant, and the invitation for the reader to methods of this paper can sample, Complexity Theory to evaluate the cost of projects to audit plans required by gated auditing standards.

Complexity Theory

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Theory was designed for understanding the cost of operating formal computer algorithms, the same tool is appropriate for analyzing behavioral problems and the cost of their solution. Since auditors have long used flowcharts to describe the movements of documents and the procedures for taking actions, [11], we can apply formal analysis to this specification.

Complexity Theory is discussed in section 2. In section 3 of this paper, we show that a part of the accounting and auditing problem embodied in the FCPA is NP Complete, and hence we can expect many internal control systems to be intractable. That is, using current methods, establishing system adequacy is not possible and in all probability, it may be an absolutely intractable problem. In section 4 we demonstrate that a subset of the FCPA is tractable, that is, it can be solved in a reasonable amount of time.

Finally, section 5 presents several policy implications, some other uses of the method of analysis, and an interpretation of the application to the FCPA. The FCPA is an interesting example of the application of Complexity Theory to Accounting. However, there are many additional issues in Accounting for which Complexity Theory is relevant, and they provide motivation for the reader to examine the methods of this paper carefully. For example, Complexity Theory could be used to evaluate the cost of proposed additions to audit plans required by newly promulgated auditing standards.

Complexity Theory

Complexity Theory is the study of how difficult it is to solve a class of problems in terms of the time it may take to solve a problem in that class. It is an area featuring some deep mathematical results. Within the scope of this paper, however, we are concerned only with some relatively ele-

mentary notions. The primary consideration is that there are classes of problems, or groups of related problems, whose complexity is on the same order. A common and important example of such a class is the set of problems that can be solved in polynomial time, and is denoted PT. The measure of complexity of a class of problems is derived in the following manner. First, find a solution algorithm that solves the class of problems. For example, there are solution algorithms to linear programming problems. The complexity of the problem is then identified as the operations cost (generally measured in computing time) of this solution algorithm. The best or lowest cost algorithm possible is used as the complexity measure. An example of this class of problems is the set of problems "find 2 times x" for any x.

PT is formally defined in the following manner. A class of problems is in PT if

there exists a polynomial $\sum_{k=1}^n a_k x^k$, so

that any member of the class of problems

of size S may be solved in $\sum_{k=1}^n a_k S^k$

steps. The size of a problem is roughly the size of the input. In the example of calculating $2x$, the input is x and the size of the input would be the number of digits in x . Thus 5 is of size 1, and 2.37 is of size 3. "Find $2x$ " is a polynomial problem that is, in PT, because it can be done in roughly $2S$ steps where S is the size of the input.

The central notion of Complexity Theory is that of a reducibility. A class of problems C is reducible to a class of problems K, if a solution to the problems in K yields solutions to the problems in C with a little extra computation. "Little" is formally defined within Complexity Theory, but for our purposes it is sufficient to think of little as "little relative to the cost of

solving K." If C is reducible to K, then problem C is considered no harder than K, and K is said to be "C-hard." That is, K is as hard as C, within a small factor.

It is more interesting to consider hardness in terms of classes of problems. For example, consider the class of problems "Find $2x$ " for input x , and call this K. Let C be the class of problems "find $2x+1$ " for input x . It is clear that, given a problem in C (find $2n+1$), we may choose a problem in K (find $2n$) that is associated with it. If we can solve the problem in K, i.e., find $2n$, then it is relatively easy to solve the problem in C, by adding one. This motivates the following definition: A class of problems K is said to be C-hard, for class C, if for any problem c in C, there is a problem k in K such that the solution of k yields the solution of the associated c with a little extra computation. Again, this extra computation is little with respect to the cost of solving k , and is generally taken to be polynomial time (depending on context). In particular, for our example involving find $2x$ and find $2x+1$, "find $2x$ " is "find $2x+1$ "-hard.

Intuitively, it would seem that "find $2x$ " is not as hard as "find $2x+1$," however, adding one is so inconsequential that the problems are considered equally hard.

A special case of C-hard problems is the notion of C-complete problems. A class of problems K is C-complete if K is C-hard and K is a subset of C. K is C-complete if K is a subset of C that is as hard to solve as C itself. Examples of PT-complete problems are very complicated; so we will look at another class of problems, which will be relevant to the problem this paper addresses.

The class NP, for nondeterministic polynomial time problems, is a peculiar class of surprising importance. A set of problems C is in NP if there is a poly-

nomial $\sum_{k=1}^n b_k x^k$ such that given a pro-

posed solution to a problem c in C, one can check to determine that the proposed solution to c is truly a solution, and it will take

no more than $\sum_{k=1}^n b_k S^k$ steps to estab-

lish whether the proposal is in fact a solution, where S is the size of c . This does not mean that we can necessarily find a solution to c in polynomial time, only that if we think we have a solution, we can verify it in polynomial time.

A classic example of an NP problem, and one that is known to be NP-complete, is the truth table satisfiability problem. Given a boolean expression involving boolean variables x_1, \dots, x_n (either true or false) and the connectives "not" (\sim) "and" ($\&$), and "or" (\vee), one asks if there is an assignment of truth or falsity to the variables such that an entire expression is true. If a truth assignment is given, then whether the expression is satisfied may be checked in linear time, i.e., in PT. Given n variables, its truth may be checked in rn steps, for some constant r . However, the only known way to actually solve the problem is to expand the truth table, and since there are 2^n possible assignments, and hence things to check, the best known way to solve the problem is in exponential time (2^n). Thus an NP problem may be intractable in requiring exponential time to obtain a definitive solution, while given a proposed solution, that solution may be checked in polynomial time.

This is an excellent example of an NP-complete problem also, because it is in NP, and known to be NP hard. At the time of this writing, *all* solution methods for NP hard problems require exponential time, or 2^n steps. Consequently, if we can show that the accounting problem solution called for by the FCPA is an NP hard problem, it will require exponential time to solve at the time of this writing. At the current level of

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computing capability, problems even of reasonable intractable. It is our beliefs of internal control; the sense employed at the FCPA would require tractable problems.

Exponential time regarded as intractable reasons (see [7]). Computationally fall by some multiplicity by half every few years. a budget B , the size S problem that can be $S = B = 2^S$, or size $S = \log_2 B$ doubled, $2B = 2(2^S) = 2^{S+1}$ increases to $S+1 = \log_2(2B)$ the budget by halving would result in a feasible increased in size by only of one. Consequently, w solve exponential time realistic size. Therefore, are rightfully considered

However, if a problem time (PT) it can be solved in n steps, for some n . In this case, the budget from $B = S$ to $S+1 = 2B$, or an increase in problem size by the multiplicity $\sqrt{2}$. Therefore, as computation grows multiplicatively, feasible problems grow multiplicatively. Intractable problems are growing very rapidly. As capacity grows, and consequently expect to eventually solve intractable, or not feasible problems are tractable.

It is important to note a class of problems is intractable force every member of the class is intractable. In fact, problems will generally be tractable of the problems of large

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problem c in C , one can show that the proposed solution, and it will take

$b_n S^k$ steps to establish

that the proposed solution is in fact a solution of c . This does not necessarily find a solution in polynomial time, only that if we have a solution, we can verify it in

polynomial time. A problem is NP-complete if it is both a member of an NP problem and NP-hard.

NP-completeness is a satisfiability problem. An expression involving variables x_1, \dots, x_n (either true or false) and connectives "not" (\sim), "and" (\wedge), "or" (\vee), "if...then" (\rightarrow), "if and only if" (\leftrightarrow), one asks if there is an assignment of truth or falsity to the variables such that the entire expression is satisfied. If an assignment is given, then the question is whether the expression is satisfied. This is a decision problem, i.e., in PT. Given n variables, one can check in polynomial time whether a given assignment satisfies the expression. However, to actually solve the problem, i.e., to find the truth table, and since there are 2^n possible assignments, and checking each one is in exponential time, the problem may be intractable. The best known way to solve an NP-complete problem is in exponential time. The problem is intractable, while given a proposed solution, one can check that solution in polynomial time.

A classic example of an NP-complete problem is the Traveling Salesman Problem, also, because it is in NP, and NP-hard. At the time of writing, no polynomial time solution methods for NP-complete problems are known. They require exponential time, or polynomial time, if we can show that a problem is intractable. A problem solution called for an NP-hard problem, it will take exponential time to solve at the current level of technology. At the current level of

computing capability, exponential problems even of reasonable size are regarded as intractable. It is our belief that many problems of internal control analysis are large in the sense employed above and thus the FCPA would require solution to intractable problems.

Exponential time problems are regarded as intractable for the following reasons (see [7]). Computing costs generally fall by some multiplicative factor, say by half every few years. This means that on a budget B , the size S of an exponential problem that can be solved is given by $B = 2^S$, or size $S = \log_2 B$. If the budget is doubled, $2B = 2(2^S) = 2^{S+1}$, then the size increases to $S+1 = \log_2(2B)$. Thus, doubling the budget by halving computing costs would result in a feasible problem which is increased in size by only an additive factor of one. Consequently, we cannot expect to solve exponential time problems of any realistic size. Therefore, NP-hard problems are rightfully considered intractable.

However, if a problem is in polynomial time (PT) it can be solved in roughly S^n steps, for some n . In these cases a doubling of the budget from $B = S^n$, results in $(\sqrt[n]{2} S)^n = 2B$, or an increase in the feasible problem size by the multiplicative constant $\sqrt[n]{2}$. Therefore, as computing costs drop multiplicatively, feasible problem sizes grow multiplicatively. Thus, the size of feasibly solvable polynomial time problems is growing very rapidly as computing capacity grows, and consequently we can expect to eventually solve them. For this reason, NP hard problems are considered intractable, or not feasible, while PT problems are tractable.

It is important to note that the fact that a class of problems is intractable does not force every member of that class to be intractable. In fact, problems of small size will generally be tractable. Moreover, some of the problems of large size will also be

tractable. However, there will be problems whose cost of solution is not feasible, even given advances in computing machinery. This is occasionally referred to as "worst case cost." Thus, showing that general compliance with a strict interpretation of the FCPA requires solution of an NP-Complete class of problems does not mean all firms will have internal control systems that are unauditable. Small firms will remain auditable. Moreover, large firms with very simple internal control systems will also be auditable. However, there will be organizational structures that satisfy the FCPA, but where the system is complex and the cost to prove compliance unbearable. The effect of the law on these firms will be considered in the conclusion to this paper. In general, we can expect one effect of the FCPA to be alterations of the structure of firms toward simpler, auditable systems. The unspecified costs of the changes may be in lost operating efficiency.

As previously noted, in this paper we shall prove formally two important results. First, that the FCPA requests that accountants solve an NP-hard problem, and therefore, in general, requires too much of the profession. NP-hard problems of the size of internal control systems are likely to be intractable. Second, that certain aspects of the collusive activities problem of firms are PT problems, and therefore may be quite feasibly solved. The collusive activities issue provides an example of a feasibly solvable problem within the context of Complexity Theory. The reader should not feel that the extensive mathematics of Complexity Theory are employed merely to prove the theorems of this paper. Complexity Theory is the only tool currently available to evaluate the cost of proposed analysis, which does not depend on analyzing a model. Difficulty with Markovian models and simulations arises out of the necessary choice of the particular model or

system to be examined. Complexity Theory is general in nature, applying to any model having similar characteristics. Consequently, it is certain to enjoy more applications in the years to come. The applications of this paper, while possessing some inherent interest, are also interesting as examples of cost estimation in a human setting. The results of this paper are intuitively plausible, but require complexity analysis for a formal proof.

A General Internal Control Problem is NP-Complete

Unfortunately, the federal government does not specify exactly what must be shown in order to be in compliance with the FCPA. Therefore, we and the courts, must interpret the act in order to model what must be shown, and hence describe exactly what problem is NP-complete.

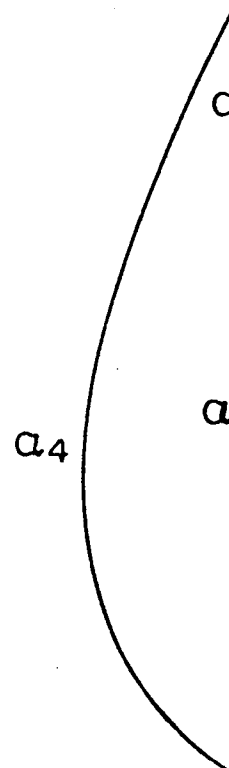
The following problem description is a very simplified situation encountered in accounting control settings. If it is NP-hard, it should not be too difficult an intuitive jump to the conclusion that more complex and realistic internal control problems are also NP-hard. We represent the agents of a firm by a set of vertices, or points. The potential information flows and actions of agents are then described by directed arrows between these points. These arrows are grouped into classes $\alpha_1, \dots, \alpha_k$ depending on the type of transaction they represent. This formalism leads us to employ what is called a directed multigraph, or DMG. A path in a DMG is defined by a set of arrows such that the first arrow ends where the second arrow begins, and in general the m^{th} arrow ends where the $m+1^{\text{st}}$ arrow begins.

Figure 1 is an example of a DMG for simple inventory requisition system. The list of symbols with their interpretations are:

- a. user of a good (production)
- b. authorization agent
- c. inventory record keeping department
- d. production control department
- e. warehouse
- α_1 inventory requisition
- α_2 authorization report
- α_3 inventory release to production
- α_4 inventory release report
- α_5 production control report
- α_6 production control report
- α_7 inventory and production reconciliation report

The intended action and control process is as follows:

1. Agent a, e.g., a production line foreman, requisitions inventory for use in the production process, α_1 ;
2. Agent b, e.g., the plant manager, authorizes the release of inventory to production, α_2 ;
3. The authorization report, α_2 , is made available to agents e and c;
4. Agent e, e.g., a warehouse supervisor, relies on this authorization when releasing inventory, α_3 , for use by agent a, the originating source of the requisition;
5. Agent c, e.g., an inventory accounting clerk, maintains an accounting of all authorized requisitions;
6. Agent e in addition to releasing inventory to the production processes, also provides an inventory release report, α_4 , to agent c. Agent e will also periodically count the physical inventory. Physical protection of the inventory is assumed to be very strong at the warehouse site.
7. Agent c matches inventory release reports, α_4 , and authorized requisitions.



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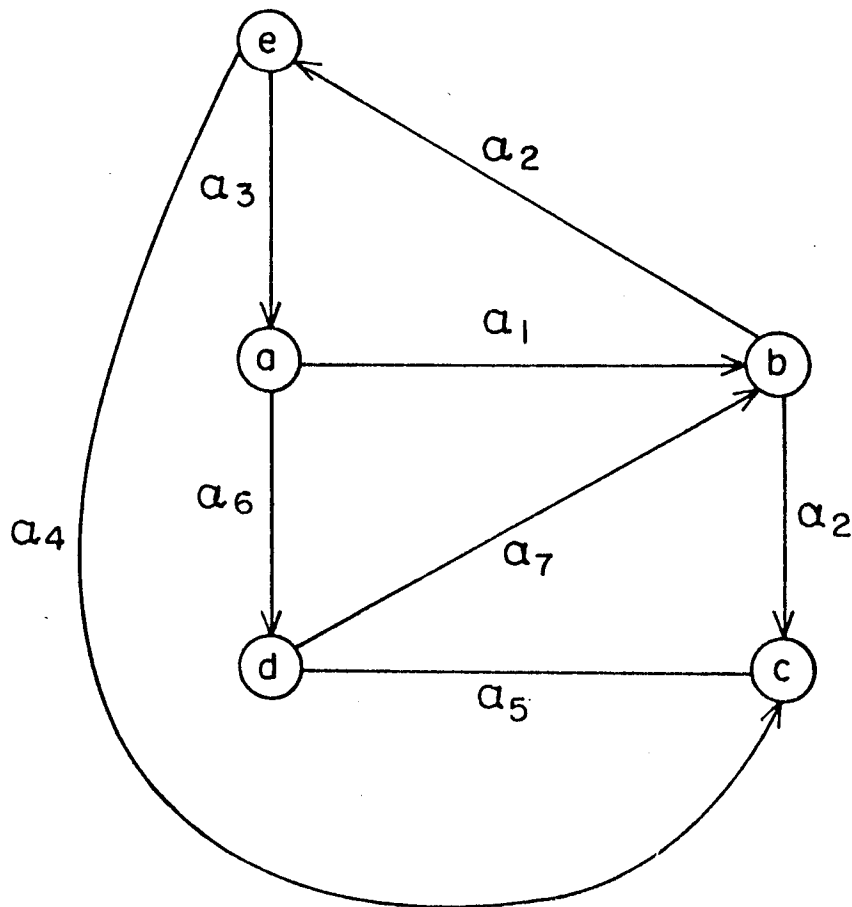
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Figure 1
 Inventory Requisition System



tions, α_2 , and forwards these data to agent d in the form of an inventory control report, α_3 ;

8. Agent a prepares a production control report, α_6 , and forwards this to agent d;
9. Agent d, e.g., an inventory control clerk, prepares an inventory and production reconciliation report, α_7 , and provides this to agent b;
10. Agent b will consider this report in making future authorization decisions.

When the system operates as intended, control over inventory use is quite strong. The internal controls and accounting process assure a continuing monitoring of the actions of the agents in charge of firm resources. This is, of course, a simplified process and control system. Nevertheless, it is sufficiently complex to serve our purposes.

For each agent in the process illustrated there exists the possibility of errors or irregularities in the performance of their functions. It may be useful to consider several possibilities. Suppose that agent c makes a series of errors in reporting to agent d. Agent d is in a position to identify these errors on the basis of data provided by agent a. If agent a uses inventories for his/her personal use, agent d is in a position to discover this fact based on the comparison of the production and inventory control reconciliation. Agent e is constrained by physical warehouse controls and the potential of an external audit not explicitly modelled.

One can certainly envision potential failures in the system that could go on for greater or lesser time periods without discovery. An expansion of the system description and restated DMG would be required to overcome these problems. It is worth a moment of reflection to consider

the eventual size of the DMG needed to represent even a fairly simple firm-wide internal control system. As the complexity of the firm increases it will become more difficult for the internal control systems designer to "envision" all of the possible control combinations necessary to assure that no internal control failure can occur. Yet in its simplicity this appears to be the requirement placed on the accountant by the FCPA, i.e., firm-wide assurance of accurate internal control over errors and irregularities. "Reasonable assurance" to be sure, but reasonable assurance in a large, complex system.

The DMG of Figure 1 makes it fairly clear that, when operating as designed, agent a cannot requisition inventory, α_1 , and take delivery, α_3 , of inventory from agent e without appropriate authorization, α_2 , from agent b. The DMG description of this process is a path from a to b to e and back to a in order for the requisition to result in delivery of inventory items. One internal control failure could be characterized as agent a's ability to requisition inventory and take delivery without an authorization. In the DMG of Figure 1 it is clear that such a path does not exist. We can prove this to ourselves in this simple case by enumerating all of the paths represented. However, a DMG of any real firm would be far too complex for such an enumeration approach. In general the search for such a path requires that we locate a set of arrows, that begins at agent a and ends at agent a, but does not include agent b. DMG problems of this type are known to be NP-complete. This is shown to be the case in the Appendix.

The importance of the conclusion that internal control problems are in general NP-complete is easily seen if we recall that NP-complete problems require exponential time to solve and are thus intractable for problems of any significant size. Internal

control problems are clear problems. We are forced to consider a straightforward consideration of the FCPA, i.e., one that accepts at face value, cannot be satisfied by accountants nor effectively audited.

Perhaps the resolution of this impossible situation is that we must accept the FCPA at face value for future developments in the SEC, Courts, and Accountants to establish whether this might be possible. (See the report [6] by the General Accounting Office.) Alternatively, we must reduce or disaggregate the control problem and its related complexity to a tractable level. Heuristics that appear to permit some simplification of the problem, but at the same time even the reduced problem is intractable. This result may be intuitively obvious to many practitioners. In this case such intuition is not necessarily supportable.

A Tractable Internal Control Problem - Identification of Potential Problem Groups

Not all issues associated with internal control problems are NP-hard. Complexity Theory can also help us identify those issues that are potentially solvable and those that are intractable. Complexity Theory may prove to be a negative but also a positive contribution to the study of internal control.

A potentially collusive problem is identified in Figure 1 by the cycle that begins and ends with agent a. Examples of such cycles are the paths a-b-e-a; b-e-a-b; a-b-e-c-d-b-e-a. This cycle includes all of the agents. A cycle is a group of agents in which each agent contributes to the group activity.

of the DMG needed to fairly simple firm-wide system. As the complexity increases it will become more difficult to design internal control systems designed to detect all of the possible control failures necessary to assure that no failure can occur. Yet it appears to be the requirement of the accountant by the firm-wide assurance of accuracy over errors and irregularities to be able to assure "adequate assurance" to be able to assure in a large,

Figure 1 makes it fairly clear that the system is not operating as designed, and that the requisition inventory, α_1 , α_2 , α_3 , of inventory from the requisition inventory, α_1 , is not appropriate authorization, α_2 . The DMG description of the path from a to b to e and back to a for the requisition to receive inventory items. One possible failure could be characterized as a's ability to requisition and take delivery without an authorization. In the DMG of Figure 1 it is clear that a path does not exist. We are left to ourselves in this simple system, trying to identify all of the paths represented, a DMG of any real firm is too complex for such an approach. In general the identification of a path requires that we know the flows, that begins at agent a and ends at agent e, but does not include agent g. Problems of this type are NP-complete. This is shown in the Appendix.

In support of the conclusion that control problems are in general NP-complete, it is easily seen if we recall that control problems require exponential time and are thus intractable for any significant size. Internal

control problems are clearly large problems. We are forced to conclude that a straightforward consideration of the FCPA, i.e., one that accepts its objectives at face value, cannot be solved by accountants nor effectively audited.

Perhaps the resolution of this seemingly impossible situation is that we should not accept the FCPA at face value. It remains for future developments in the Congress, SEC, Courts, and Accounting profession to establish whether this might be the case. (See the report [6] by the General Accounting Office.) Alternatively, we may be able to reduce or disaggregate the internal control problem and its related DMG to a tractable level. Heuristics currently exist that appear to permit some effective segmentation of the problem, however, at this time even the reduced problems are intractable. This result may have been intuitively obvious to many practicing accountants. In this case such intuition is theoretically supportable.

A Tractable Internal Control Problem - Identification of Potential Collusive Groups

Not all issues associated with internal control problems are NP-hard. Complexity Theory can also help us identify those issues that are potentially solveable as well as those that are intractable. Thus, Complexity Theory may prove to have not only a negative but also a positive role to play in the study of internal control issues.

A potentially collusive group can be identified in Figure 1 by locating a path that begins and ends with a particular agent. Examples of such cyclic paths are the paths a-b-e-a; b-c-d-b, and a-b-e-c-d-b-e-a. The last path includes all of the agents. A cycle identifies a group of agents in which each agent can contribute to the group action in such a

way as to make a fraudulent act appear to be a valid act. Thus, each agent in the cycle is necessary to the collusive fraud.

The last cycle noted above includes the complete set of agents and is not particularly interesting in that it represents a complete override of a larger system in which case agents a through e might benefit by a collusion within the larger group.

Collusive group b-c-d-b is capable of affecting only the record keeping activity and might benefit each other in covering errors, but it appears to have no access to firm resources and is thus not of as much potential interest as collusive group a-b-e-a. Agents a, b, and e, when acting together can access firm assets and by their actions maintain basic accounting records in such a way as to preclude identification of a defalcation without an audit activity external to the described system. Agents c and d are not a part of the cycle and thus not necessary to the collusion. Agents a, b and e could maintain consistent accounting while stealing inventories by agreeing among themselves as to the content of information flows α_2 , α_4 and α_6 . As a result agents c and d will observe no discrepancies in the reporting process. From an audit perspective, identification of these potential collusive groups would be helpful in directing audit effort. An audit of agent e's physical inventories and a comparison to the accounting data are not detectable within the system. An external audit agent is necessary for detection.

The identification of cycles such as those discussed above is called the cycle problem in DMG's. Problems of this type are solvable in polynomial time. This may be formally shown by reducing these cycles into the two literal conjugate normal form satisfiability problem, known to be in PT. The critical point here is that polynomial time problems are more likely to be cur-

rently tractable, and as computing capacity grows, more likely to become tractable than NP-hard problems. Thus, while accountants and auditors cannot solve the general problem posed by the FCPA they can feasibly attack the issue of collusive group identification.

Based on a generalization of the internal control description process utilized in this paper, the authors are implementing a system capable of detecting potential collusive groups. The approach adopted makes use of the latest computer technology to seek solutions and is the topic of separate papers. (See [3] and [4].)

Policy Implications

There are several aspects of this paper meriting attention. First, in order to evaluate the meaning of a law, a formal interpretation must be given. In order to define a term precisely, a model must be constructed such that meaning may be identified within the context of the model. Directed multigraphs were employed to model a subset of the FCPA, as it related to internal control issues. In order to demonstrate the utility of this representation, one internal control failure was identified with sets of transactions lacking proper authorizations. While this does not encompass the full realm of internal accounting failures, the problem was still sufficiently complex to be generally intractable.

This has important ramifications for policy analysts. In this case, the law requires analysis which is theoretically possible but not feasible in practice. Because the analysis is not feasible, in general, the cost of the analysis must outweigh any benefits derived from the law. However, an alternate view may be proposed which is perhaps more plausible. The results of this paper show that certain organizational structures are sufficiently

difficult to analyze so that verification of internal control properties is not feasible.

This does not necessarily mean that all firm structures present intractable analysis problems, but only that analysis of some of the structures will be intractable. Consequently we may expect to see two methods of evading the feasibility problem. First, accountants will interpret the law in less than a strict manner. This may be necessary for accountants who may bear some of the costs for internal control failures in the future. Second, accountants may require firms to institute certain structural alterations so that the firm becomes easier to analyze. That is, the accountants may require the firm to take on a certain structure that is "feasibly certifiable" rather than analyze the firm as it stands. This is a logical move on the accountants' part, because by requiring a certifiable organizational structure, they shift the costs of internal accounting failures away from themselves. Perhaps these are naive alternatives. However, by subtle means we believe that both conditions do occur. Accountants and auditors do interpret to suit their needs and firm structures are altered, however slightly, to accommodate the new circumstances.

This is not likely to result in efficient firm organization. Firms have evolved over the years in ways responding to the business environment, and obviously the firm views its own structure as efficient [8]. To require changes in firm design merely because the firm isn't feasibly analyzable demands changes for uncertain gains. A given firm may satisfy the FCPA, but is such a labyrinth that certification of this compliance is not feasible. As a result, an efficient firm may become less efficient simply to establish that it complies with a law which it may have complied with originally.

Significantly, we did demonstrate that one interpretation of the collusion problem

is tractable. This shows that both tractable and infeasible tasks may be handled by the analytic techniques of this paper. This paper serves several purposes. It justifies the accountant's decision in providing a formal proof that the problem is indeed an unbearable burden.

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roduces new methods of cost/benefit analysis for proposed regulations and their effect on institutions. Third, and even more importantly, it provides a variety of tools and concepts for the analysis and design of internal control systems.

Figure 2

G. A Hamilton path is marked by the double arrows.

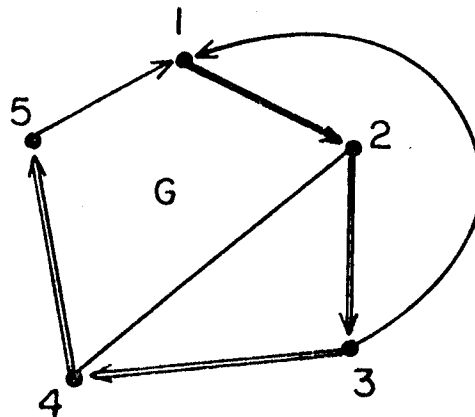


Figure 3

H. Note the arrows are named according to their destinations.

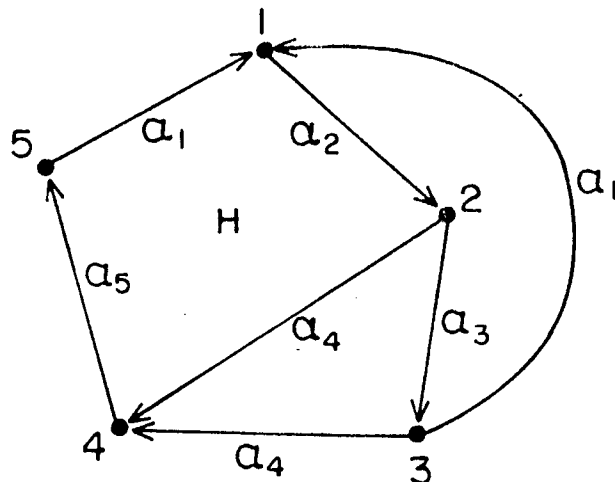
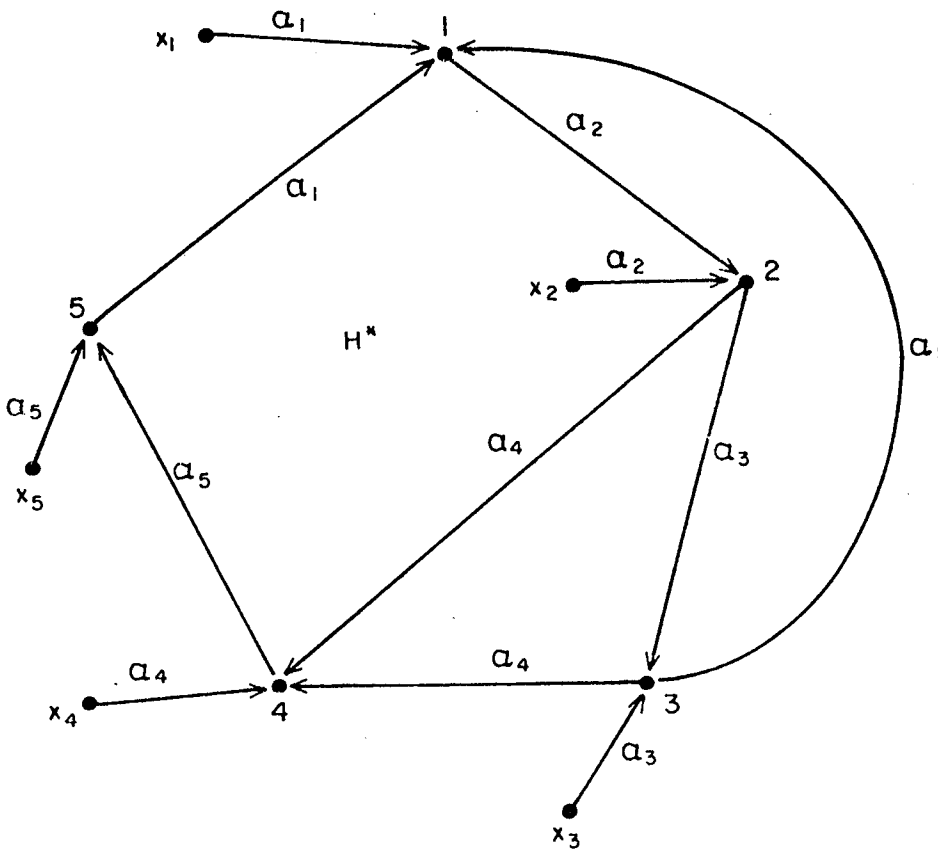


Figure 4
Expansion of H to H*



A DMG problem has a (see Figure 2) and a set of problem: "is there a path in the arrows in Γ ?", is the 1 To see that the set of DM NP-complete, we show first NP, and second that a pro be NP-complete, the Hamil lem [7] may be reduced to t lem.

The Hamilton path follows. Given a directed points with arrows between points, all the arrows of the Hamilton path problem as path which visits each point

Proposition: The DMG complete. **Proof:** (i) To see problem is in NP, observe the posed solution to a DMG must only follow the path ar the members of Γ are used e no others are used. This req of steps at most about twice path, and hence is linear in t and thus polynomial.

(ii) To reduce a given H problem with graph G to a D we must construct a multie and a set Γ so that the DMG solution if and when the H problem has a solution. So rected graph with k vertice Figure 3) be a multigraph wi rows as G, except that every at the i^{th} vertex is an α_i type for each vertex i in H, defin

APPENDIX

A DMG problem has a DMG graph G (see Figure 2) and a set of arrows Γ . The problem: "is there a path in G using exactly the arrows in Γ ?", is the DMG problem. To see that the set of DMG problems is NP-complete, we show first that DMG is in NP, and second that a problem known to be NP-complete, the Hamilton path problem [7] may be reduced to the DMG problem.

The Hamilton path problem is as follows. Given a directed graph (a set of points with arrows between some of the points, all the arrows of the same type), the Hamilton path problem asks if there is a path which visits each point exactly once.

Proposition: The DMG problem is NP-complete. **Proof:** (i) To see that the DMG problem is in NP, observe that given a proposed solution to a DMG problem, we must only follow the path and observe if all the members of Γ are used exactly once and no others are used. This requires a number of steps at most about twice the size of the path, and hence is linear in the input of Γ , and thus polynomial.

(ii) To reduce a given Hamilton graph problem with graph G to a DMG problem, we must construct a multilevel graph H^* and a set Γ so that the DMG problem has a solution if and when the Hamilton graph problem has a solution. So let G be a directed graph with k vertices. Let H (see Figure 3) be a multigraph with the same arrows as G , except that every arrow ending at the i^{th} vertex is an α_i type arrow. Finally, for each vertex i in H , define a new vertex

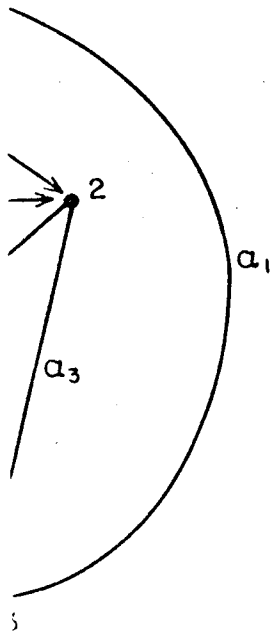
x_i , unconnected to any of those in H and connect x_i to i by an α_i arrow, for each i , $i = 1, \dots, k$. Call this resulting multigraph H^* . (See Figure 4.)

Claim: H^* has a path solving the DMG problem for $\Gamma = \{1, \dots, k\}$ if and only if G has a Hamilton path. **Proof of claim:** Suppose H^* has a DMG path for Γ . Clearly this path visits each vertex, as it uses one of each α_i , and α_i ends in vertex i . Suppose it starts with an arrow α_i from x_i to i (clearly no $x_i \rightarrow i$ arrow is in the middle, as nothing leads to x_i), delete this arrow, and the resulting path is a Hamilton path for G (as it visits each point once, starting as i).

So suppose the DMG path does not involve a $x_i \rightarrow i$ type arrow, for any i . We then have a Hamilton cycle, because the path uses each type of arrow, and hence visits each point once and ends at its origin. Therefore, if we delete any arrow from the path, we have a Hamilton path.

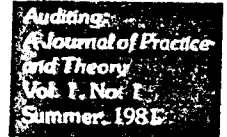
Conversely, suppose we have a Hamilton path for G . To construct a DMG path for H^* , take the initial point i of the Hamilton path and add the arrow $x_i \rightarrow i$. Since this path visits each point once, it must possess exactly one arrow of each type, and is thus a DMG path, as visiting a point i means using an i type arrow.

Finally observe that the size of H^* is about twice the size of G , thus we have added linear (and hence polynomial) overhead to solving the Hamilton path problem via reduction to DMG. Consequently, the DMG problem is at least as hard as the Hamilton path problem, as desired.



REFERENCES

- [1] Arthur Anderson & Co., *A Guide for Studying and Evaluating Internal Accounting Controls*, (AA, 1978).
- [2] Arthur Young & Co., *Evaluating Accounting Controls: A Systematic Approach*, (AY, 1980).
- [3] Bailey, A.D., Jr., Gerlach, J., McAfee, R.P. and Whinston, A.B., "TICOM II - The Internal Control Language," *Florida Symposium on Internal Controls*, A. Rashad Abdel-khalik, Ed., (University of Florida Press, forthcoming, 1981a).
- [4] _____, "Internal Accounting Controls in the Office of the Future," *IEEE Computer* (May 1981).
- [5] Deloitte, Haskins & Sells, *Internal Accounting Control, An Overview of the DH&S Study and Evaluation Techniques*, (DH&S, 1979).
- [6] General Accounting Office, "Impact of Foreign Corrupt Practices Act in U.S. Business," (GAO, 1981).
- [7] Machtey, M., and Young, P., *An Introduction to the General Theory of Algorithms*, (Elsevier North-Holland, Inc., 1978).
- [8] Maher, M.W., and Mautz, R.K., "Internal Control in the Business Curriculum," *The Auditors Report* (Winter, 1981), p. 6.
- [9] Mautz, R.K., et. al., *Internal Control in U.S. Corporations: The State of the Art* (Financial Executives Research Foundation, 1980).
- [10] Peat, Marwick, Mitchell & Co., "SEC Chairman Seeks to Remove Anxieties Arising from Foreign Corrupt Practices Act," *Executive Newsletter*, (January 26, 1981), p. 12.
- [11] Skinner, R.M., and Anderson, R.J., *Analytical Auditing: An Outline of the Flow Chart Approach to Audits*, (Pitman, 1966).



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THE Commission on responsibilities (CAR) was created in 1974 as an independent study by the Board of Directors of the American Institute of Certified Public Accountants (AICPA). CAR was charged t