Animal Spirits

By PETER HOWITT AND R. PRESTON MCAFEE*

This paper constructs a stationary rational-expectations equilibrium in which an extraneous random variable, called animal spirits, causes fluctuations in unemployment. The model assumes costly matching in the labor market and a thin-market externality in the output market that makes the profitability of hiring depend positively on the number of firms hiring. The equilibrium does not rely on any effect of expected inflation on labor supply. It is also stable under learning; Bayesian updating induces convergence to the equilibrium with positive probability even if people start with no definite belief that animal spirits affect the profitability of hiring. (*JEL* E32)

Until recently, modern research in business-cycle theory has been guided almost entirely by the view that fluctuations in the overall level of economic activity are the result of exogenous shocks to the fundamental conditions of a dynamically stable economic system. By this view, booms and recessions are attributable to random changes in variables such as the availability of profitable investment opportunities, the propensity to save, the stance of macroeconomic policy, population, international terms of trade, and the distribution of demand, either contemporaneous or lagged, or to the arrival of information signalling such changes. The view was first given formal expression by Ragnar Frisch (1933) and Eugen Slutzky (1937), but goes back at least to William S. Jevons (1884), to whom sunspots were a real fundamental, and is embodied today in the "new classical" models of Robert E. Lucas (1975) and others and in the equilibrium real-business-cycle theories of such authors as Finn E. Kydland and Edward C. Prescott (1982).

Several recent authors have begun to revive a competing view of business cycles, according to which fluctuations would occur even if fundamental conditions were to remain unchanged over time. This view has two variants. The first sees fluctuations as endogenous, resulting from a failure of the economic system to settle down to a stationary state even in the absence of shocks. For example, in the nonlinear multiplier-accelerator models of Richard M. Goodwin (1951) and J. R. Hicks (1950) full-employment equilibrium is unstable, but various floors and ceilings prevent activity from exploding or imploding and thus keep it fluctuating indefinitely. Earlier writers accepted this view as the most natural, without the benefit of much formal analysis, perhaps by analogy to the variety of cycles exhibited in nature. Indeed, the very terminology of business cycles implies the view that cycles, not rest, constitute the natural motion of the economic system. This variant has recently been formalized by authors whose nonlinear systems exhibit either periodic equilibria (Jean-Michel Grandmont, 1985) or chaos (Richard H. Day, 1982).

The second variant of the competing view attributes fluctuations to random waves of optimism and pessimism that are unrelated to fundamental conditions. This view is often attributed to John Maynard Keynes, who argued that entrepreneurs' animal spirits were an important determinant of investment, but it could equally be attributed to

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John Stuart Mill or F. A. von Hayek, and goes back at least as far as Henry Thornton (1802). It has recently been revived by the work of Costas Azariadis (1981), Michael Woodford (1988), and others on what are commonly (but misleadingly, in the light of Jevons's views) referred to as sunspot equilibria. This work has shown that animal spirits can have an influence even in rational-expectations equilibria. People condition their expectations about, say, the rate of return to investment on some extraneous random variable. Belief that this variable signals changes in the rate of return can be self-fulfilling. When it increases unexpectedly (i.e., when animal spirits rise) people undertake actions that will, on average, make the equilibrium return rise as expected.

This paper constructs a model of a rational-expectations animal-spirits cycle. The objective is twofold. The first is to show by construction that such cycles do not depend upon the assumption, common to all the recent sunspot literature, that fluctuations in aggregate employment are driven by fluctuations in the expected rate of inflation, which induce workers to vary the amount of labor offered for sale. Consider, for example, the model of Azariadis (1981). It is a simple overlapping-generations model of money, in which the demand-for-money/supply-of-labor schedule is backward bending. The expectation of a high rate of inflation can be self-fulfilling, because it induces an increase in the quantity of money demanded, which causes an instantaneous fall in the price level (assuming a constant money supply), which will indeed be followed by a high rate of inflation when the price level returns to normal. Likewise, the expectation of a low rate of inflation can be self-fulfilling. In an animal-spirits equilibrium, the young are induced to supply much labor when expected inflation is high and to supply little when it is low.

Woodford (1988) has shown that animal-spirits cycles are not restricted to the overlapping-generations model and that they can be achieved without the possibly objectionable assumption of a backward-bending labor-supply schedule. However, even in this model, variations in the expected rate of inflation play an important role in determining variations in the amount of labor supplied.

There are well-known empirical reasons to doubt that the aggregate supply of labor is highly responsive to the return from work. There are particularly good reasons to doubt that the elasticity of that response is sufficiently negative to be consistent with the Azariadis model. Furthermore, nothing in the literature on the (non)superneutrality of money suggests that expected inflation has an empirically significant effect on that return.[1] Thus, the case for animal-spirits cycles will likely remain unconvincing to all but a small number of economists until the cycles can be exhibited in models with empirically more plausible propagation mechanisms.

The present paper attempts to do just that. It exhibits an animal-spirits cycle in a model with no role assigned to expected inflation. Instead, it follows Peter A. Diamond's (1984) suggestion of deriving the cycle from transaction externalities that produce multiple stationary equilibria. The particular model used is a variant on one that we have developed elsewhere (Howitt and McAfee, 1988).[2]

We model animal spirits as an exogenous random variable that follows a two-state Markov process, switching between high and low. When spirits are high, firms expect a high level of employment, and hence a high level of aggregate demand. The prospect of

[1]For summaries of evidence on these issues, see Olivier J. Blanchard and Stanley Fischer (1989 pp. 181, 193, 341–6).

[2]Models with multiple stationary equilibria arising from transaction externalities can be embraced in the general canonical coordination-failure model of Russell W. Cooper and Andrew John (1988). Animal-spirits equilibria do more, however, than randomize between otherwise unconnected static equilibria of such models. In the example of this paper the rational anticipation of random oscillations in future activity levels induced by animal spirits affects the determination of current activity levels and is in turn affected by the anticipation of oscillations yet to come, ad infinitum. The dynamic multiple-equilibrium models of Nobuhiro Kiyotaki (1988) and Philippe Weil (1989) last only two periods and have no stochastic disturbances.

high demand reduces the expected cost of contacting a potential customer in the output market. This encourages firms to hire more vigorously, thus validating the original expectation. Likewise, when spirits are low, firms expect low employment and low aggregate demand and are induced to fulfill these expectations by hiring less.

Employment does not just oscillate between two static equilibrium levels. Instead, it follows a random mixing of two separate autoregressive processes.[3] When spirits are high, employment rises asymptotically toward a high stationary level, according to a first-order linear difference equation. When spirits are low, it falls asymptotically toward a low stationary level, according to a different first-order difference equation.

The downturns of this cycle have much in common with Keynes's (1936) account of depressions in the *General Theory*. Not only is the downturn driven by a fall in animal spirits, but the positive feedback between the actual and expected levels of employment acts like a multiplier process. What makes firms reduce hiring is not the variation in some market price or expected market price, but a nonprice signal, in the form of higher selling costs. What makes total work-effort fall is not the voluntary decision of workers to sell less labor, but the increased difficulty of finding job offers when firms have cut back on hiring. The fact that wage and price rigidities play no role in the downturn is also consistent with the intention of the *General Theory*.

The second objective of the paper is to address one of the most troubling questions concerning animal-spirits cycles, namely, how would people ever acquire the beliefs underlying such a cycle. More precisely, suppose that people were not endowed with rational expectations but instead formed their expectations through some plausible adaptive learning scheme. Would the sequence of temporary equilibria converge to one in which animal spirits mattered, or would people learn to ignore the fundamentally extraneous information?

This question is the sort addressed by the literature on convergence to rational expectations (e.g., Roman Frydman and Edmund S. Phelps, 1983). In this case, the rational expectations are the ones underlying the animal-spirits cycle. Recent work by George W. Evans (1989) shows that in many models learning will not converge to a rational-expectations equilibrium with extraneous conditioning variables.[4] We show that this is not the case in the present model under Bayesian learning. Starting from an initial situation in which everyone has diffuse priors, beliefs will converge on those of the rational-expectations animal-spirits cycle with positive probability.

I. The Model

There is a fixed number of identical firms and a unit mass of identical workers per firm. They interact in two markets—for output and labor—using pure inside money which plays no explicit role in the analysis. Time is discrete. Each period there are δ new workers born (per firm), where $\delta \in (0,1)$, and each existing worker has the constant probability δ of dying. Firms live forever. Everyone has the same additive linear preferences over lifetime consumption, with the subjective discount factor $\beta \in (0,1)$.

Each newborn worker enters the labor market and begins searching for a firm. Once matched, a worker will bargain with the firm, and the bargain will result in a lifetime employment contract, requiring the worker to devote his entire endowment of labor services to producing output for the firm and giving the worker the fraction w each period of the current value of the match.

[3] This time-series representation is similar to the one for which James D. Hamilton (1989) found evidence in U.S. quarterly GNP data. Michael D. Boldin (1989) found that a representation of GNP and unemployment based on the model below fits U.S. data quite well.

[4] Woodford (1990) shows that convergence occurs in the Azariadis model, but Evans (1989) shows that this result is not robust. If people begin looking at another extraneous conditioning variable, then their expectations will not converge to the original equilibrium.

Work begins the period after the match has been made, at which time the worker withdraws from further search.

The value of a match in period t is $fg(n_t)$, where f (>0) is the constant marginal product of labor, n_t is aggregate employment per firm, and $g(n_t)$ is 1 minus the cost per unit of selling output. This cost depends negatively on n_t because of a transaction-externality. As in Diamond (1982) or Howitt (1985), higher employment means higher aggregate demand and more willing customers, and this is assumed to reduce the marginal cost of contacting a willing customer. Firms are small enough that they treat n_t as given.

ASSUMPTION 1: *For each* $n \in [0,1]$, $g(n)$ *is continuously differentiable, with* $0 \leq g(n) \leq 1$ *and* $g'(n) > 0$.

Let λ_t be the expected value to the firm of hiring an additional worker at t. It follows that:

$$(1) \quad \lambda_t = \beta(1-\delta)E_t[f(1-w)g(n_{t+1}) + \lambda_{t+1}]$$

where E_t denotes the firm's expectation conditional on information at t.

The matching technology works as follows. At any date t there will be a mass $1-n_t$ of unemployed workers per firm, all searching for a firm. (In equilibrium, employed workers cannot gain from searching.) A firm that wishes to contact the fraction θ_t of these searchers must pay a cost $c_t\theta_t$ in the form of output used in the recruiting process. The cost parameter c_t is an independent and identically distributed random variable whose mean is $c > 0$. The realization of c_t is not known at the time of the hiring decision. Thus, the firm's expected recruiting cost per contact is $(E_t c_t)/(1-n_t)$.[5] Since all contacts result in a hire, this is also the expected recruiting cost per worker hired. It is an increasing function of aggregate employment because of an externality implicit in the above discussion. Specifically, an increase in the number of searching workers allows the firm to make more contacts at no extra cost.

This matching technology can be derived from more primitive assumptions, as in Howitt and McAfee (1987), by assuming that each searching worker goes through space in a random direction and at a constant speed, until encountering the recruiting net cast by a firm, and that the recruiting nets cover the fraction θ_t of space.

For simplicity we make the special assumption that the technology allows only two possible values of θ_t, 0 or h, where h is a positive fraction. Each firm will compare the cost per hire with the benefit. Thus it will set

$$\theta_t = \begin{cases} 0 & \text{if } \lambda_t < (E_t c_t)/(1-n_t) \\ h & \text{if } \lambda_t > (E_t c_t)/(1-n_t). \end{cases}$$

Since all firms are identical, employment will obey

$$(2) \quad n_{t+1} = \begin{cases} n^{\mathrm{L}}(n_t) \equiv (1-\delta)n_t & \\ \qquad \text{if } \lambda_t < (E_t c_t)/(1-n_t) \\ n^{\mathrm{H}}(n_t) \equiv (1-\delta)[n_t + h(1-n_t)] & \\ \qquad \text{if } \lambda_t > (E_t c_t)/(1-n_t) \end{cases}$$

where n_0 is given by history. Given any expectation mechanism, equations (1) and (2) constitute the equilibrium conditions of the model.

II. Perfect Foresight

As we have shown (Howitt and McAfee, 1988) in a continuous-time version of this model, there may be many perfect-foresight equilibria starting from the same initial employment level. Here we focus on two of them, corresponding to what Diamond and Drew D. Fudenberg (1989) call the pessimistic and optimistic paths. Along the pessimistic path, everyone correctly believes that there will be no recruiting, and employment falls gradually to zero through attrition. Along the optimistic path, everyone correctly believes that all firms will actively

[5]Since E_t is not a rational expectation when firms are learning, therefore $E_t c_t$ is not always equal to c.

recruit at all times, and employment asymptotically approaches a stationary value $n^{\mathrm{H}} \in (0,1)$.

The possibility that each of these paths might constitute a perfect-foresight equilibrium starting from the same level of employment arises because of the transaction externality. Starting from $n_0 < n^{\mathrm{H}}$, if everyone expects the economy to follow the optimistic path they will expect the cost of selling output to fall continually, because $g' > 0$. The prospect of low future costs will have a positive effect on the current value of hiring, through (1). This may be enough to justify current hiring, according to (2), and hence to fulfill the optimistic expectation. On the other hand, if everyone expects the economy to follow the pessimistic path, then the prospect of high future costs resulting from low future employment might depress the value of hiring by enough to fulfill the pessimistic expectation.

More formally, define

$$n^{\mathrm{H}} \equiv \frac{(1-\delta)h}{(1-\delta)h+\delta} \in (0,1).$$

For any $n \in [0,1]$, define

$$\lambda^{\mathrm{L}}(n) \equiv \sum_{i=1}^{\infty} \beta^i (1-\delta)^i f(1-w) g\left[(1-\delta)^i n\right]$$

and

$$\lambda^{\mathrm{H}}(n) \equiv \sum_{i=1}^{\infty} \Big\{ \beta^i (1-\delta)^i f(1-w) \times g\left[n^{\mathrm{H}} + (1-\delta)^i (1-h)^i (n - n^{\mathrm{H}})\right]\Big\}.$$

Then, $\lambda^{\mathrm{L}}(n)$ [respectively, $\lambda^{\mathrm{H}}(n)$] would be the value of hiring when employment equalled n if people had perfect foresight and the economy were on the pessimistic [optimistic] path. If

$$(3) \quad \lambda^{\mathrm{L}}(n) < c/(1-n) < \lambda^{\mathrm{H}}(n) \quad \text{for all } n \in [0, n^{\mathrm{H}}]$$

then for all $n_0 \in [0, n^{\mathrm{H}}]$, both the pessimistic and optimistic paths are perfect-foresight equilibria; that is, they satisfy (1) and (2), with E_t interpreted as the identity operator in (1) and $E_t c_t = c$ in (2).

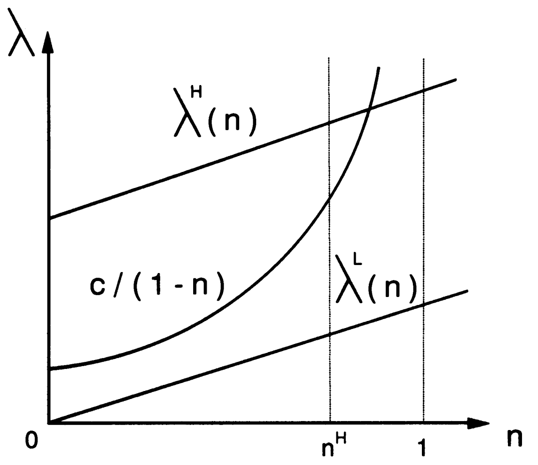

FIGURE 1. AN EXAMPLE IN WHICH BOTH THE PESSIMISTIC AND OPTIMISTIC PATHS ARE PERFECT-FORESIGHT EQUILIBRIA

Condition (3) is illustrated in Figure 1. The crucial assumption needed to satisfy the condition is that there is enough of an external economy of scale in the output market, operating through the transaction technology. That is, g' must be large enough that the shadow value λ lies everywhere below the cost curve $[c/(1-n)]$ when employment is expected to fall along the pessimistic path, but everywhere above it when employment is rising along the optimistic path. This would be impossible in the absence of the thin-market externality, because if $g' = 0$ then λ^{L} and λ^{H} would coincide and be horizontal. Ricardo J. Caballero and Richard K. Lyons (1989) present evidence for strong external economies of scale in U.S. manufacturing, although of course this does not imply that the economies are strong enough to satisfy the analogue to (3) in a realistic model of the U.S. economy.

An example satisfying (3) is given by

$$g(n) = n/(1+\varepsilon)$$

$$\varepsilon > 0 \qquad h = 1 \qquad 1 > \delta > (1+\varepsilon)/4$$

$$f = \frac{4c\left[1-\beta(1-\delta)^2\right]}{(1-w)\beta(1-\delta)^2}.$$

This is verified in the Appendix. The example is obviously robust, since small perturbations in any parameter or in g will leave the strict inequalities (3) satisfied.

III. Animal-Spirits Cycles

An animal-spirits cycle is a cycle in which employment stays in $[0, n^{\mathrm{H}}]$ and switches randomly between the optimistic and pessimistic paths. Which path the economy is on at t depends upon the value of an extrinsic random variable $s_t \in \{\mathrm{L}, \mathrm{H}\}$, which we call animal spirits. When spirits are high ($s_t = \mathrm{H}$), every firm recruits. When spirits are low ($s_t = \mathrm{L}$), none recruits. Animal spirits follow a two-state Markov process with transition matrix:

$$A = \begin{bmatrix} a^{\mathrm{LL}} & a^{\mathrm{LH}} \\ a^{\mathrm{HL}} & a^{\mathrm{HH}} \end{bmatrix} = \begin{bmatrix} 1 - a^{\mathrm{L}} & a^{\mathrm{L}} \\ a^{\mathrm{H}} & 1 - a^{\mathrm{H}} \end{bmatrix}$$

where a^{L} (a^{H}) is the probability of change when spirits are low (high). These probabilities are independent of the random hiring cost c_t.

Formally, for any $n_0 \in [0, n^{\mathrm{H}}]$, the animal-spirits cycle (ASC) is the random sequence $\{n_{t+1}\}_0^\infty$ satisfying

$$n_{t+1} = n^i(n_t) \quad \text{if } s_t = i \qquad i = \mathrm{L}, \mathrm{H}$$

for all $t = 0, 1, \ldots$. It is straightforward to verify that the ASC remains in $[0, n^{\mathrm{H}}]$ forever.

Let $\hat{\lambda}(n, i, a)$ be the continuous real-valued function on $\mathrm{K} \equiv [0, n^{\mathrm{H}}] \times \{\mathrm{L}, \mathrm{H}\} \times [0, 1]^2$, defined by the functional equation:[6]

$$(4) \quad \hat{\lambda}(n, i, a) = \beta(1-\delta)\left[f(1-w) g(n^i(n)) + \sum_{j=\mathrm{L}}^{\mathrm{H}} a^{ij} \hat{\lambda}(n^i(n), j, a) \right]$$

where $a = (a^{\mathrm{L}}, a^{\mathrm{H}})$. This function defines the rational expectation of the value of hiring when current employment is n, the current state of animal spirits is i, the transition probabilities are a, and the economy is following an ASC. If

$$(5) \quad \hat{\lambda}(n, \mathrm{L}, a) < c/(1-n) < \hat{\lambda}(n, \mathrm{H}, a) \qquad \text{for all } n \in [0, n^{\mathrm{H}}]$$

then the ASC will be a rational-expectations equilibrium; that is, it will satisfy (1) and (2) with E_t interpreted as the mathematical expectation conditional on (n_t, s_t).

A robust example satisfying (5) can be constructed, because in the degenerate case where $a = 0$ the rationally expected value of hiring when spirits are high (low) is exactly the perfect-foresight value on the optimistic (pessimistic) path. That is, $\hat{\lambda}(n, i, 0) = \lambda^i(n)$. Thus, any example satisfying (3) satisfies (5) when $a = 0$. Since $\hat{\lambda}$ is continuous, it will also satisfy (5) for some strictly positive values of a.[7]

In short, even though expectations are driven by animal spirits, they can be rational. People may rationally anticipate the waves of optimism and pessimism that keep employment fluctuating forever. What drives the boom is the expectation of rising aggregate demand, not expectations of inflation. The crucial assumption sustaining this cycle as an equilibrium is that there exists enough of an economy of scale in the output market, working through a thin-market transaction externality, that condition (3) is satisfied, not the assumption of a

[6]The existence, uniqueness, and continuity of the function $\hat{\lambda}$ are ensured by the contraction-mapping theorem (see e.g., Thomas J. Sargent, 1987 pp. 343–4). Specifically, the right-hand side of (4) defines a contraction mapping with modulus $\beta(1-\delta)$ on the complete space of continuous functions λ: $\mathrm{K} \to \mathbb{R}_+$ with metric $d(\lambda, \lambda') \equiv \max\{|\lambda(x) - \lambda'(x)| \text{ subject to } x \in \mathrm{K}\}$. The same statements apply to the functions $\tilde{\lambda}$ described by (9).

[7]Thus, an ASC can be a rational-expectations equilibrium when probabilities of change $(a^{\mathrm{L}}, a^{\mathrm{H}})$ are small enough, whereas the argument of Azariadis (1981) shows the same in an overlapping-generations model when those probabilities are large enough. Accordingly, our model need not imply the high-frequency oscillations in employment that would tend to be exhibited by the Azariadis model. It is also worth noting that our existence argument is quite different from that of Woodford (1988), which involves randomizing expectations in the neighborhood of a stationary state where the perfect-foresight dynamics would yield indeterminacy.

backward-bending labor-supply or savings function.[8]

IV. Learning Animal Spirits in a Simplified Model

A rational-expectations interpretation of mob psychology may seem incongruous. It also begs the question of how anyone would ever arrive at such peculiar expectations (see Evans, 1989). Both of these considerations suggest modeling expectations according to a more adaptive scheme that does not endow firms ab initio with beliefs consistent with the model. This section presents a simplified version of the model and shows that beliefs can converge to the rational expectations of the ASC under Bayesian learning even though everyone's priors are diffuse. The next section extends the analysis to the full model.

Intuitively, spurious correlation between the animal-spirits signal s_t and recruitment-cost c_t can cause firms to condition the hiring intensity θ_t on s_t. This spurious correlation eventually disappears; however, the data produced continue to exhibit a correlation between s_t and θ_t which does not vanish, and, indeed, becomes perfect.

The model is simplified by having workers live for exactly two periods with certainty, instead of having the geometric distribution of lifetimes. Each period, there will be a unit mass of young workers looking for a job, so the recruiting cost per worker will be c_t. The hired workers work next period. The rest drop out of the economy. The value of hiring a worker will be either $\lambda^{\mathrm{H}} \equiv \beta f(1-w)g(h)$, if everyone hires, or $\lambda^{\mathrm{L}} \equiv \beta f(1-w)g(0)$ if no one hires, with $\lambda^{\mathrm{L}} < \lambda^{\mathrm{H}}$. (Mixing equilibria are assumed away.) The random cost parameter c_t is either c^{H} or c^{L}, with $c^{\mathrm{H}} < c^{\mathrm{L}}$. (Mnemonically, the low-cost c^{H} leads to high employment.)

The firm needs to know four probabilities:

$$p^s \equiv \Pr\left(c_t = c^{\mathrm{H}} | s_t = s\right) \qquad s = \mathrm{L},\mathrm{H}$$

$$q^s \equiv \Pr\left(\theta_t = h | s_t = s\right) \qquad s = \mathrm{L},\mathrm{H}.$$

In a rational-expectations equilibrium, $p^{\mathrm{L}} = p^{\mathrm{H}} = \bar{p}$, where

$$c = (1-\bar{p})c^{\mathrm{L}} + \bar{p}c^{\mathrm{H}}. \tag{6}$$

An animal-spirits equilibrium is one in which $q^{\mathrm{L}} = 0$ and $q^{\mathrm{H}} = 1$. An optimistic equilibrium is one in which $q^{\mathrm{L}} = q^{\mathrm{H}} = 1$, and a pessimistic equilibrium is one in which $q^{\mathrm{L}} = q^{\mathrm{H}} = 0$. All three equilibria exist if $\lambda^{\mathrm{L}} < c < \lambda^{\mathrm{H}}$, in which case the rationally expected cost of hiring will be less than the benefit if everyone is hiring, but greater than the benefit if no one is hiring. As in the model of the previous sections, the crucial assumption is that there is a large enough external economy of scale; without $g' > 0$ there would be no gap between λ^{L} and λ^{H}.

Out of rational-expectations equilibrium, the firm knows the values of λ^{L}, λ^{H}, c^{L}, and c^{H}, but not the probabilities (p^s, q^s). It believes that these probabilities are constant over time. This belief is correct in the case of the p's, and will be correct eventually with respect to the q's as well if the economy gets to a rational-expectations equilibrium. Each firm has identical beliefs. It starts with independent diffuse priors over each of the probabilities (i.e., with a uniform subjective distribution on [0,1] on each probability). It then updates these priors each period using Bayes's Rule.

Let $(p_t^{\mathrm{L}}, p_t^{\mathrm{H}}, q_t^{\mathrm{L}}, q_t^{\mathrm{H}})$ denote the expected value of (p,q) according to the beliefs at date t. Then, in each period, the observation of (c_t, θ_t) will provide useful information in updating $(p_t^{\mathrm{H}}, q_t^{\mathrm{H}})$ if $s_t = \mathrm{H}$, or $(p_t^{\mathrm{L}}, q_t^{\mathrm{L}})$ if $s_t = \mathrm{L}$. However, the independence of priors implies that the observation provides no information that can be used for updating the probabilities associated with the state that was not observed. It follows from well-known results on Bayesian

[8]Although λ is the price of an asset (hired labor), the model yields no implications concerning the "excess volatility" of asset prices (Stephen F. LeRoy and Richard D. Porter, 1981). In animal-spirits equilibrium the dividend-stream $(1-w)fg(n_t)$ associated with the asset varies more than can be accounted for by variability in the economy's fundamentals, but the asset price does not vary more than can be accounted for by variability in the dividends. Instead, according to (4), the standard expected present-value relationship holds between price and dividend.

estimation of the parameter of a binomial distribution that

$$(7) \qquad (p_t^s, q_t^s) = \left(\frac{m_t^s + 1}{\tau_t^s + 2}, \frac{k_t^s + 1}{\tau_t^s + 2} \right)$$

$$s = \mathrm{L}, \mathrm{H} \quad t = 1, 2, \dots$$

where τ_t^s is the number of times up to and including $t-1$ at which the state s has been observed, m_t^s is the number of those times at which the cost c^{H} has also been observed, and k_t^s is the number of times when other firms have recruited in state s.[9]

Consider a firm at the beginning of period t, having observed $s_t = s$. The firm's expected profits are

$$\pi(p_t^s, q_t^s) = q_t^s \lambda^{\mathrm{H}} + (1 - q_t^s) \lambda^{\mathrm{L}} - [p_t^s c^{\mathrm{H}} + (1 - p_t^s) c^{\mathrm{L}}]$$

if it hires, and zero if not. Define the recruitment region,

$$\mathrm{R} = \{(p,q) | \pi(p,q) > 0\}$$

and the nonrecruitment region

$$\mathrm{N} = \{(p,q) | \pi(p,q) < 0\}.$$

The firm will recruit if $(p_t^s, q_t^s) \in \mathrm{R}$, and not if $(p_t^s, q_t^s) \in \mathrm{N}$.

Assume

$$(8) \qquad c^{\mathrm{H}} < \lambda^{\mathrm{L}} < c < \lambda^{\mathrm{H}} < c^{\mathrm{L}}.$$

This set of assumptions is equivalent to $(\bar{p}, 1) \in \mathrm{R}$, $(\bar{p}, 0) \in \mathrm{N}$, $(1,0) \in \mathrm{R}$, and $(0,1) \in \mathrm{N}$. The first two ensure that an animal-spirits cycle is a rational-expectations equilibrium.

[9]The learning scheme in (7) is almost identical to the non-Bayesian scheme of estimating (p^s, q^s) by the sample mean $[(m_t^s/\tau_t^s), (k_t^s/\tau_t^s)]$. The latter is the scheme suggested by Margaret M. Bray (1983) and is a special case of least-squares learning. Both schemes are examples of the general adaptive process specified by Paul R. Milgrom and D. John Roberts (1990). What distinguishes the present paper is not the learning rule, but the demonstration that it can lead to an animal-spirits equilibrium.

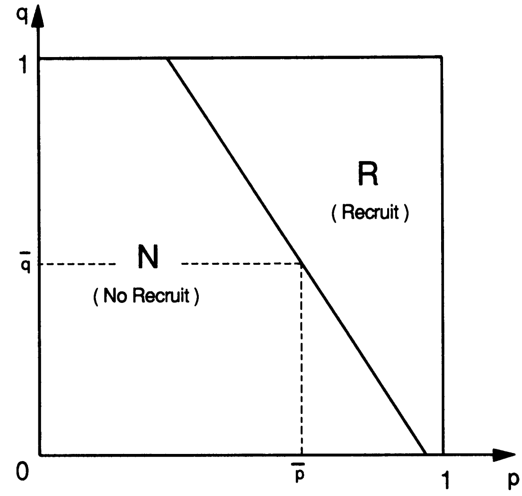

FIGURE 2. FIRMS RECRUIT WHEN THERE IS A HIGH ENOUGH COMBINATION OF p, THE PROBABILITY THAT RECRUITING COSTS WILL BE LOW, AND q, THE PROBABILITY THAT OTHER FIRMS WILL BE RECRUITING

That is, if firms have correct beliefs about the probability p that $c_t = c^{\mathrm{H}}$, then they recruit if and only if the others recruit. The second two assumptions ensure that there are values of p such that the expectation concerning other firms is irrelevant. If p is close enough to 1 (0), then a firm recruits (does not recruit) regardless of q. The value $\bar{q}$, defined by $\pi(\bar{p}, \bar{q}) = 0$, is important in what follows. From assumption (8),

$$\bar{q} = \frac{c - \lambda^{\mathrm{L}}}{\lambda^{\mathrm{H}} - \lambda^{\mathrm{L}}} \in (0,1).$$

Assumption (8) and the regions R and N are illustrated in Figure 2.

The Bayesian updating rule allows a simple geometric description of the evolution of beliefs, as given in the following lemma.

LEMMA 1: *If* $(p_t^s, q_t^s) \in \mathrm{R}$ *and* $s_t = s$, *then*

(i) (p_{t+1}^s, q_{t+1}^s)

$$= \begin{cases} \left(\frac{\tau+2}{\tau+3}\right)(p_t^s, q_t^s) + \left(1 - \frac{\tau+2}{\tau+3}\right)(0,1) & \text{if } c_t = c^{\mathrm{L}} \\ \left(\frac{\tau+2}{\tau+3}\right)(p_t^s, q_t^s) + \left(1 - \frac{\tau+2}{\tau+3}\right)(1,1) & \text{if } c_t = c^{\mathrm{H}}. \end{cases}$$

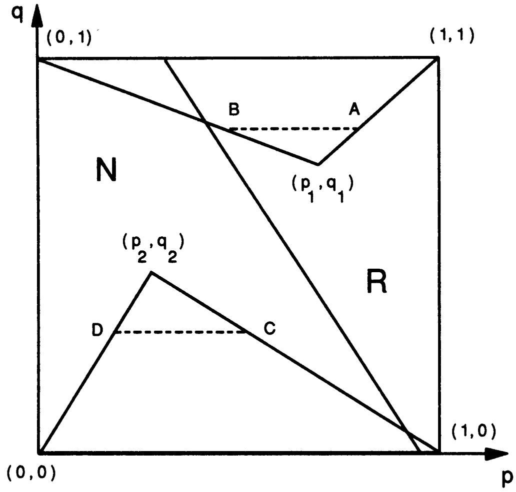

FIGURE 3. STARTING AT $(p_1,q_1) \in$ R, THE SUBSEQUENT STATE MUST LIE ON THE LINE SEGMENT CONNECTING (p_1,q_1) TO (1,1) AT A (IF $c_t = c^{\mathrm{H}}$) OR TO (0,1) AT B (IF $c_t = c^{\mathrm{L}}$); SIMILARLY, STARTING AT $(p_2,q_2) \in$ N, THE SUBSEQUENT STATE IS ON THE LINE SEGMENT CONNECTING (p_2,q_2) TO (1,0) AT C (IF $c_t = c^{\mathrm{H}}$) OR (0,0) AT D (IF $c_t = c^{\mathrm{L}}$)

Similarly, if $(p_t^s,q_t^s) \in$ N *and* $s_t = s$,

(ii) (p_{t+1}^s,q_{t+1}^s)

$$= \begin{cases} \left(\frac{\tau+2}{\tau+3}\right)(p_t^s,q_t^s)+\left(1-\frac{\tau+2}{\tau+3}\right)(0,0) & \text{if } c_t = c^{\mathrm{L}} \\ \left(\frac{\tau+2}{\tau+3}\right)(p_t^s,q_t^s)+\left(1-\frac{\tau+2}{\tau+3}\right)(1,0) & \text{if } c_t = c^{\mathrm{H}}. \end{cases}$$

PROOF:

Suppose $(p_t^s, q_t^s) = [(m+1)/(\tau+2), (k+1)/(\tau+2)] \in$ R. Since all firms recruit, $q_{t+1}^s = (k+2)/(\tau+3)$ and

$$p_{t+1}^s = \begin{cases} \dfrac{m+2}{\tau+3} & \text{if } c_t = c^{\mathrm{H}} \\ \dfrac{m+1}{\tau+3} & \text{if } c_t = c^{\mathrm{L}} \end{cases}$$

and (i) follows immediately; (ii) is similar.

Lemma 1 shows that posterior beliefs on (p^s,q^s), after a new observation $s_t = s$ arises, always lie on a line segment connecting the prior beliefs with one of the four corners, as illustrated in Figure 3.

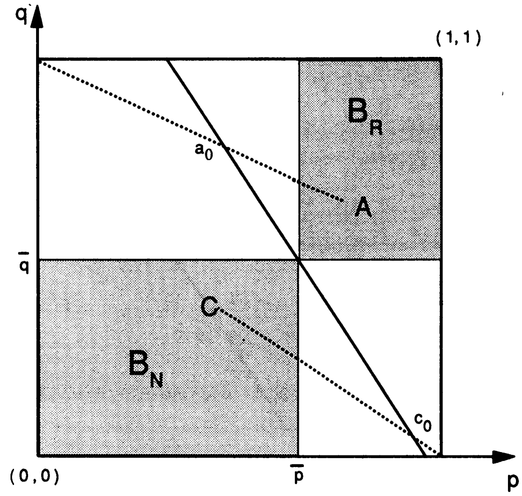

FIGURE 4. ILLUSTRATION OF $\mathrm{B_R}$ AND $\mathrm{B_N}$: NOTE THAT, TO LEAVE R STARTING AT A $\in \mathrm{B_R}$, IT IS NECESSARY THAT $p_t^s < \bar{p}$ SOMEWHERE ALONG THE WAY; SIMILARLY, TO LEAVE N STARTING AT C $\in \mathrm{B_N}$, $p_t^s > \bar{p}$ IS NECESSARY

Remark: Beginning at any (p,q) in the interior of $[0,1]^2$, there is a positive probability of changing regions. Suppose $(p_t^s,q_t^s) \in$ R. Referring to Figure 3, subsequent beliefs, if $s_t = s$ and $c_t = c^{\mathrm{L}}$, are at a point like B. A long enough string of c^{L}'s, therefore, will force beliefs into N. Similarly, beginning in N, a long string of c^{H} observations will eventually send beliefs into R. By the same reasoning, there is a positive probability of entering any given neighborhood of the corner (1,1) or (0,0).

By the law of large numbers, the estimate p_t^s eventually goes to the true parameter value $\bar{p}$. Define two rectangles:

$$\mathrm{B_R} = \{(p,q): \bar{p} < p \le 1, \bar{q} \le q \le 1\} \subset \mathrm{R}$$

$$\mathrm{B_N} = \{(p,q): 0 \le p < \bar{p}, 0 \le q \le \bar{q}\} \subset \mathrm{N}.$$

These rectangles (shown in Fig. 4) have the following property. To exit R and enter

N, starting at $(p_t^s, q_t^s) \in B_R$, at some point along the way $p_{t_0}^s < \bar{p}$. Similarly, to exit N starting at $(p_t^s, q_t^s) \in B_N$, $p_{t_0}^s > \bar{p}$ at some time $t_0 > t$. It turns out that, with positive probability, this does not occur. To show this, we need a technical lemma.[10]

LEMMA 2: *Let $x_t \in \{0,1\}$ be an independent and identically distributed sequence of Bernoulli random variables, with* $\Pr\{x_t = 1\} = p$ *and* $a > p$. *Let* $y_T = (1/T)\sum_{t=1}^{T} x_t$. *Then,* $\Pr\{(\forall n)(y_n \le a)\} > 0$.

THEOREM 1: *There is a positive probability that*

$$(p_t^{\mathrm{H}}, q_t^{\mathrm{H}}) \to (\bar{p}, 1) \text{ and}$$

$$(p_t^{\mathrm{L}}, q_t^{\mathrm{L}}) \to (\bar{p}, 0).$$

That is, an animal-spirits cycle arises and persists forever.

PROOF:

Note that $(p_t^{\mathrm{H}}, q_t^{\mathrm{H}})$ and $(p_t^{\mathrm{L}}, q_t^{\mathrm{L}})$ are, by construction, independent of each other. Thus, we need only show a positive probability that $(p_t^{\mathrm{H}}, q_t^{\mathrm{H}}) \to (\bar{p}, 1)$ and a positive probability that $(p_t^{\mathrm{L}}, q_t^{\mathrm{L}}) \to (\bar{p}, 0)$.

Note that by the Remark there is a positive probability that $(p_t^{\mathrm{H}}, q_t^{\mathrm{H}}) \in B_R$ at some point. Given a point $(p, q) \in B_R$, let (p_0, q_0) represent the point on the line segment connecting (p, q) to $(0, 1)$ satisfying $\pi(p_0, q_0) = 0$. This point is denoted as a_0 in Figure 4. Thus, to leave R starting at (p, q), it is necessary that $p_T \le p_0 < \bar{p}$ at some time $T > t$; but we know from Lemma 2 that

$$\Pr\{(\forall T > t)\, p_T > p_0\} > 0.$$

Thus, there is a positive probability that, once in B_R, beliefs remain in R. This, in turn, forces $q_t^{\mathrm{H}} \to 1$. By the law of large numbers, $p_t^{\mathrm{H}} \to \bar{p}$.

Proof that $(p_t^{\mathrm{L}}, q_t^{\mathrm{L}}) \to (\bar{p}, 0)$ with positive probability is analogous.

On a more intuitive level, what this theorem shows is the possibility that firms may be led into the pattern of hiring when spirits are high, because of an initial correlation between high spirits and a low cost of recruiting, and not hiring when spirits are low, because of an initial correlation between low spirits and high costs. Eventually they come to learn that costs are independent of spirits; but meanwhile they have learned that aggregate hiring depends upon spirits and that therefore the benefit to hiring depends upon spirits. The strategic complementarity imparted by thin-market externalities makes that lesson self-reinforcing.

Clearly what is needed for this initial correlation to lead ultimately to an animal-spirits cycle with high probability is a set of circumstances in which (a) people's initial beliefs put them on or near the margin

[10] Lemma 2 follows easily from the law of the iterated logarithm (Patrick Billingsley, 1986 p. 151) and independence of the random variables. The law guarantees the existence of an integer N such that $\Pr\{(\forall n \ge N)(y_n < a)\} > 0$. By independence,

$$\begin{aligned}
&\Pr\{(\forall n)(y_n \le a)\} \\
&\quad \ge \Pr\{x_1 = \dots = x_N = 0 \text{ and } (\forall n \ge N)\, y_n \le a\} \\
&\quad = \Pr\{x_1 = \dots = x_N = 0\} \\
&\qquad \times \Pr\{(\forall n \ge N)(y_n \le a) | y_n = 0\} \\
&\quad \ge \Pr\{x_1 = \dots = x_N = 0\} \\
&\qquad \times \Pr\{(\forall n \ge N)(y_n \le a)\} \\
&\quad = (1-p)^N \times \Pr\{(\forall n \ge N)\, y_n \le a\} > 0.
\end{aligned}$$

The first inequality follows from event inclusion; the equality follows by the definition of conditional probability; and the second inequality follows by independence and the fact that

$$y_n \ge (1/n) \sum_{t=N}^{n} x_i$$

which is y_n given that $x_1 = \dots = x_N = 0$. We thank Paul Milgrom and Philip Reny for assistance with this argument.

between recruiting and not recruiting, and (b) once they have decided to recruit (not recruit) it takes an unusually bad (good) run of draws on recruiting cost to persuade them to change their minds. The most extreme instance of (a) and (b) occurs in the limiting case where $\lambda^{\mathrm{H}} = c^{\mathrm{L}}$ and $\lambda^{\mathrm{L}} = c^{\mathrm{H}}$, in which case the probability of ending up in an animal-spirits cycle can be as large as $\frac{1}{2}$.

More specifically, in that case, the line dividing the two regions R and N in Figure 1 is the diagonal from (0,1) to (1,0). Since $(p_0^s, q_0^s) = (\frac{1}{2}, \frac{1}{2})$, the initial point for each state of spirits will lie on this dividing line, where firms will be indifferent between hiring and not hiring. The random choice made to break this initial indifference will decide whether firms hire or not in that state forever, since in that case there is zero probability of leaving either R or N. The probability of being eventually in an animal-spirits equilibrium is just the probability that the indifference is broken differently in the two states. Indifference will be broken in either state the first time firms choose to recruit and a low cost is drawn, or choose not to recruit and a high cost is drawn. Therefore, assuming that the random indifference-breaking mechanism is unbiased and independent across states, the probability of an animal-spirits equilibrium is $2\bar{p}(1-\bar{p})$, which can be as large as $\frac{1}{2}$.

V. Learning Animal Spirits in the Full Model

Consider now the full model in which workers are subject to the constant death rate δ. The intrinsic labor-market dynamics are more complicated than in the simplified model because the effect of the hiring intensity θ_t on employment and, hence, on the current costs and benefits of hiring are not confined to a single period. Nevertheless, the stability analysis of the previous section goes through with only minor modifications.

Assume again that people know everything about the economy except for the probabilities (p^s, q^s), which people believe to be a vector of constants, and that their expectations of the (p,q)'s, evolve according to (7). The value of λ_t implied by these beliefs [the value consistent with (1)] is $\tilde{\lambda}(n_t, s_t, q_t^{\mathrm{L}}, q_t^{\mathrm{H}})$, where $\tilde{\lambda}$ is the solution to the functional equation

$$(9)\quad \tilde{\lambda}(n,s,q^{\mathrm{L}},q^{\mathrm{H}}) = \beta(1-\delta)\Bigg\{ f(1-w)\big[q^s g(n^{\mathrm{H}}(n)) + (1-q^s) g(n^{\mathrm{L}}(n))\big] + \sum_{j=\mathrm{L}}^{\mathrm{H}} \Big(a^{sj}\big[q^s \tilde{\lambda}(n^{\mathrm{H}}(n), j, q^{\mathrm{L}}, q^{\mathrm{H}}) + (1-q^s)\tilde{\lambda}(n^{\mathrm{L}}(n), j, q^{\mathrm{L}}, q^{\mathrm{H}})\big]\Big)\Bigg\}.$$

Note that:

$$(10)\quad \tilde{\lambda} \text{ is increasing in } q^{\mathrm{L}} \text{ and in } q^{\mathrm{H}}$$

and

$$(11)\quad \tilde{\lambda}(n,s,0,1) = \hat{\lambda}(n,s).$$

Define the vector of probabilities: $\mathbf{b}_t \equiv (p_t^{\mathrm{L}}, p_t^{\mathrm{H}}, q_t^{\mathrm{L}}, q_t^{\mathrm{H}})$. The expected profitability of hiring in state s is

$$\pi^s(n_t, \mathbf{b}_t) \equiv \tilde{\lambda}(n_t, s, q_t^{\mathrm{L}}, q_t^{\mathrm{H}}) - \big[p_t^s c^{\mathrm{H}} + (1-p_t^s)c^{\mathrm{L}}\big]/(1-n_t).$$

The animal-spirits cycle will persist as long as $\mathbf{b}_t$ lies in the set

$$\mathrm{B} \equiv \big\{\mathbf{b} \in [0,1]^4 \,|\, \pi^{\mathrm{L}}(n,\mathbf{b}) < 0 < \pi^{\mathrm{H}}(n,\mathbf{b}) \text{ for all } n \in [0, n^{\mathrm{H}}]\big\}.$$

It follows from (5), (6), and (11) that

$$(12)\quad (\bar{p}, \bar{p}, 0, 1) \in \mathrm{B}.$$

Also, there exists $\tilde{\mathbf{b}} \in (0,1)^4$ such that $\tilde{p}^H < \bar{p} < \tilde{p}^L$ and $B^* \subset B$, where[11]

$$B^* \equiv \left\{ \mathbf{b} \in [0,1]^4 \middle| (p^L, q^L) < (\tilde{p}^L, \tilde{q}^L) \text{ and } (p^H, q^H) > (\tilde{p}^H, \tilde{q}^H) \right\}.$$

If B^* is ever entered, the animal-spirits cycle will be observed. By (7) q_t^L will be nonincreasing, and q_t^H will be nondecreasing. Therefore, $\mathbf{b}_t$ will remain in B^*, and the animal-spirits cycle will persist, for at least as long as $p_t^L < \tilde{p}^L$ and $p_t^H > \tilde{p}^H$. By Lemma 1, there is a positive probability that this will happen forever.

It remains to show that there is a positive probability of entering B^*. To this end, assume the analogue of (8):

$$(8') \quad c^H/(1-n) < \tilde{\lambda}(n, H, 0, 0) \text{ and } c^L/(1-n) > \tilde{\lambda}(n, L, 1, 1) \;\forall n \in [0, n^H].$$

Equivalently, $\pi^L(n; 0, 1, 1, 1) < 0 < \pi^H(n; 0, 1, 0, 0)$ for all $n \in [0, n^H]$. It follows from (8′) and (10) that there is a pair $(\bar{p}^L, \underline{p}^H) \in (0,1)^2$ such that

$$(13) \quad \pi^H(n; \mathbf{b}) < 0 < \pi^L(n; \mathbf{b}) \quad \text{for all } n \in [0, n^H] \quad \text{whenever } p^L < \bar{p}^L \text{ and } p^H > \underline{p}^H.$$

It follows from (7) that starting from $\mathbf{b}_0 = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, a long enough string of $s_t = L$, $c_t = c^L$ will make $p_t^L < \bar{p}^L$ in finite time. Let the string continue. By (13) and the fact that $\partial \pi^L / \partial p^H = 0$, $\theta_t = 0$ will be observed. According to (7) $(p_t^L, q_t^L) < (\tilde{p}^L, \tilde{q}^L)$ in finite time. Now let the string be succeeded by a string of $s_t = H$, $c_t = c^H$. By the same reasoning, $(p_t^H, q_t^H) > (\tilde{p}^H, \tilde{q}^H)$ in finite time. Therefore, there is positive probability that $\mathbf{b}_t \in B^*$ in finite time and positive probability that the animal-spirits cycle will persist forever.[12]

VI. Conclusion

We have presented a rational-expectations model of business cycles driven by animal spirits. The path of aggregate employment switches randomly between an optimistic path, with firms hiring, and a pessimistic path, with firms not hiring. Waves of optimism and pessimism are self-fulfilling because of a thin-market externality that makes production more profitable when others are producing a lot. The externality is assumed to work through the transaction technology but could be interpreted equally well as a production externality.

Although the model makes several very special assumptions, it also has several advantages over existing "sunspot" models. It does not rely upon any implausibly large

[11]PROOF: By the continuity of $\tilde{\lambda}$ (see footnote 6), B is open relative to $[0,1]^4$. By this and (12) there is an ε-neighborhood N of $(\bar{p}, \bar{p}, 0, 1)$ such that $N \subset B$. Define $\tilde{\mathbf{b}} \equiv (\bar{p} + \varepsilon, \bar{p} - \varepsilon, \varepsilon, 1 - \varepsilon)$ in the closure of N. Define B^* in accordance with $\tilde{\mathbf{b}}$. We just need to show that $B^* \subset B$. Take any $\mathbf{b} \in B^*$. Then, for all $n \in [0, n^H]$:

$$\begin{aligned}
\pi^L(n, \mathbf{b}) &\equiv \tilde{\lambda}(n, L, q^L, q^H) - [p^L c^H + (1 - p^L) c^L]/(1-n) \\
&< \tilde{\lambda}(n, L, q^L, q^H) - [\tilde{p}^L c^H + (1 - \tilde{p}^L) c^L]/(1-n) \quad \text{because } p^L < \tilde{p}^L \\
&\equiv \pi^L(n; \tilde{p}^L, \tilde{p}^H, q^L, q^H) \\
&< 0 \quad \text{because } (\tilde{p}^L, \tilde{p}^H, q^L, q^H) \in N
\end{aligned}$$

and, by analogous reasoning, $\pi^H(n, \mathbf{b}) > 0$.

[12]Obviously, there is also a positive probability that the economy will converge to either the optimistic or the pessimistic path (i.e., that people will learn to ignore the animal spirits). By the same token, if an ASC were disturbed by people beginning to observe a second, independent, extrinsic random variable there is a positive probability that they would learn to ignore that second variable (i.e., that the economy would remain in the original ASC). Thus, the model is immune to Evans's (1989) criticism of Woodford (1990) (see footnote 4).

effect of inflation on the voluntary supply decision of workers to make employment fluctuate or upon a perversely sloped supply relationship. Unemployment arises naturally because of the assumption of costly search and recruiting. The behavior of real wages $[wfg(n_t)]$ is procyclical, as is the behavior of productivity $[fg(n_t)]$. Employment exhibits positive serial correlation, instead of the negative correlation characteristic of the two-state Markov model of Azariadis (1981).[13]

We have also shown that the animal-spirits cycle is potentially stable under Bayesian learning. If people start with diffuse priors, there is a positive probability that accidental correlations during the early stages of learning could lead them forever into the self-fulfilling beliefs of the animal-spirits cycle.

There are other possible explanations for how people might come to use extraneous variables as leading indicators. For example, consider an agricultural economy, in which output depends on weather, which in turn is correlated with sunspots. Suppose this economy switches to manufacturing, which does not depend on weather. People will remember a correlation between output (and hence per-unit production costs) and sunspots. Under the assumptions of this paper, this correlation may persist, even though any real connection between sunspots and production has vanished. Like Pavlov's dog, who continued to display the expectation of food (salivation) when the bell rang, long after food was not forthcoming, the economy will continue to condition on a variable that is no longer correlated with any real shock. Unlike Pavlov's dog, the economy finds that its expectations continue to be fulfilled.

It is hazardous to judge the likelihood of animal-spirits cycles on the basis of a simple analysis like this. The events that lead to a perpetual cycle may appear to have low probability. However, the probability would be increased if animal spirits had a real effect on costs. An unusually high correlation early in the learning process could make people condition their selection of equilibria as well as their marginal choices on such a variable. Furthermore, there is no end to the number of potential extrinsic conditioning variables. The likelihood of a spurious correlation with at least one of them, leading to an equilibrium that conditions on it, is of course much higher than the likelihood of conditioning on any given variable.

[13] Of course, more complicated dynamics would result if reproducible capital were introduced.

Appendix

To verify the example in Section II, define

$$\varphi^i(n) \equiv \lambda^i(n)(1-n)/c \qquad i=\mathrm{L,H} \quad n\in[0,1].$$

It suffices to show that

$$\max_{[0,n^{\mathrm{H}}]} \varphi^{\mathrm{L}}(n) < 1 < \min_{[0,n^{\mathrm{H}}]} \varphi^{\mathrm{H}}(n).$$

From the definition of $\lambda^{\mathrm{L}}(\cdot)$,

$$\varphi^{\mathrm{L}}(n) = \left\{\sum_{1}^{\infty} \beta^i(1-\delta)^i f(1-w)(1-\delta)^i n/(1+\varepsilon)\right\}(1-n)/c$$

$$= \frac{f(1-w)n(1-n)\beta(1-\delta)^2}{(1+\varepsilon)c[1-\beta(1-\delta)^2]}$$

which is maximized at $n=\frac{1}{2}$. Therefore,

$$\max_{[0,n^{\mathrm{H}}]} \varphi^{\mathrm{L}}(n) \le \varphi^{\mathrm{L}}(\tfrac{1}{2})$$

$$= f\left\{\frac{(1-w)\beta(1-\delta)^2}{4c[1-\beta(1-\delta)^2]}\right\}(1+\varepsilon)^{-1}$$

$$= (1+\varepsilon)^{-1} < 1.$$

Likewise, by the definition of $\lambda^{\mathrm{H}}(\cdot)$,

$$\varphi^{\mathrm{H}}(n) = \left\{\sum_{1}^{\infty} \beta^i(1-\delta)^i f(1-w)n^{\mathrm{H}}/(1+\varepsilon)\right\}(1-n)/c$$

which is strictly decreasing in n. Since $n^{H} = (1-\delta)$,

$$\min_{[0,n^{H}]} \varphi^{H}(n) = \varphi^{H}(n^{H})$$

$$= \varphi^{H}(1-\delta)$$

$$= \frac{f(1-w)(1-\delta)\beta(1-\delta)\delta}{(1+\varepsilon)[1-\beta(1-\delta)]c}$$

$$= \left\{\frac{f(1-w)\beta(1-\delta)^{2}}{4c\left[1-\beta(1-\delta)^{2}\right]}\right\}$$

$$\times \left[\frac{1-\beta(1-\delta)^{2}}{1-\beta(1-\delta)}\right]\left(\frac{4\delta}{1+\varepsilon}\right)$$

$$= \left[\frac{1-\beta(1-\delta)^{2}}{1-\beta(1-\delta)}\right]\left(\frac{4\delta}{1+\varepsilon}\right)$$

$$> \frac{4\delta}{1+\varepsilon}$$

$$> 1.$$

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