Ec 11 Homework 1
Professor R. Preston McAfee
Winter 2005

CALTECH

1. Suppose the demand for lawn chairs is given by q_d = 60000 – 100*p*, and the supply of lawn chairs is given by q_s = 100*p*, where *p* is the price. Solve for the equilibrium price and quantity.
2. If the supply doubles (at each price, twice as much is supplied), what happens to the equilibrium price and quantity in exercise 1?
3. How will the following affect the price of fish, and why? With indirect effects (created from other markets), you should identify whether the effects are complements or substitutes. If there is no significant effect, say no effect.
    a. Discovery of bovine spongiform encephalopathy (mad cow disease) in US cattle herds
    b. Improved freezing technology lowers the cost of ice
    c. Prices of grain increase significantly
    d. Price of apples
4. Why does the supply curve give the marginal cost? Please explain at the level that of a newspaper article.
5. Caltech does not sell admission. Why not hold an auction for the right to attend Caltech?

Thumbnail Answers:

1. $q_d = 60000 – 100p = q_s = 100p$, so 200 p = 60000, p=300. q=20000.

2. $q_d = 60000 – 100p = q_s = 200p$, so 300 p = 60000, p=200. q=30000

3

a. Discovery of bovine spongiform encephalopathy (mad cow disease) in US cattle herds

fish and beef are demand substitutes, so reduction in supply of (safe) beef increases price of safe beef, demand for fish rises, and price, quantity of fish increase

b. Improved freezing technology lowers the cost of ice

cost reduction in production of fish, shifts out supply of fish, reducing price and increasing quantity

c. Prices of grain increase significantly

grain is an input to beef, increasing the price of beef which would increase demand for fish, but also possibly a substitute for fish, which would tend to increase demand for fish. This tends to increase price and quantity of fish. Since catfish are fed grain, there could be a small effect the other way.

d. Price of apples

Hard to see the price of apples having much of an effect on the fish market. Possibly a substitute for some consumer.

4. Sellers will offer their goods for sale only if the price covers the cost of the good, so that the quantity of goods offered for sale are just those for which the cost is below the price. The highest cost good offered for sale at a given price has a cost just below or equal to the price, and thus the quantity offered then is determined by the marginal cost.

5. Thought question, many possible right answers. One is: Selling only to those able to pay very high tuition selects rich, rather than smart, students, reducing the quality of Caltech graduates, injuring faculty research and running down the brand name capital of the institute. A second is: students learn a lot from other students, and thus adding a rich but not very bright student reduces the value of the education to other students, perhaps reducing their willingness to pay.

Ec 11 Homework 2
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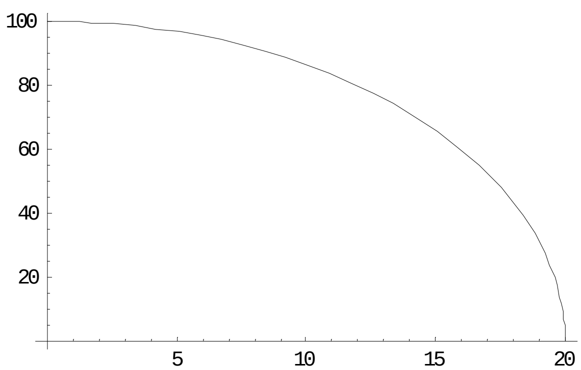
CALTECH

1. Suppose demand and supply have constant elasticity and are both increased by 1%. What happens to price and quantity?
2. Do Text exercise 2.6.1.1
3. In exercise 2.6.1.1, suppose trusses sell for $4,000 and cabinets sell for $1,000. What assignment of workers maximizes revenue? [Hint: Your answer may require "splitting" a worker across two tasks.]
4. Do Text exercise 2.6.1.2. Assume it takes the company A^2 hours to do A installations, not each individual worker.
5. Politicians are currently complaining about American jobs being transferred to India, and a decade ago they complained about American jobs moving to Mexico. Does America lose overall from buying Mexican car parts and Indian software and telephone help? Are some identifiable groups helped or hurt?
6. What is comparative advantage? Are the gains from trade with the United States larger for similar nations, or dissimilar nations?

Answers:

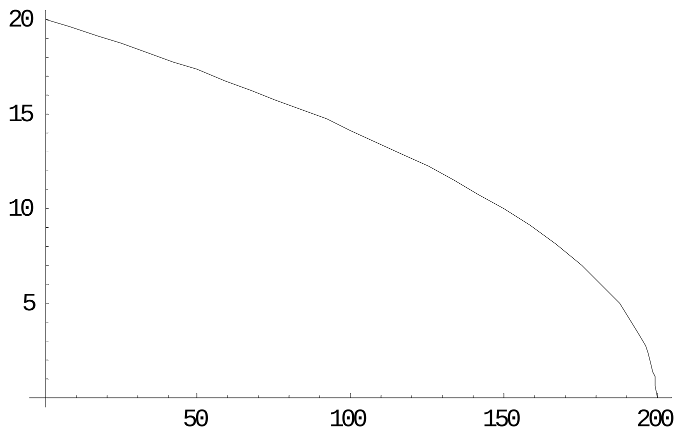
1. Using the text, demand is $q_d(p)=ap^{-\varepsilon}$ and supply function by $q_s(p)=bp^{\eta}$. The equilibrium price is $p^* = \left(\frac{a}{b}\right)^{1/(\varepsilon+\eta)}$ and quantity is $q^* = a^{\eta/(\varepsilon+\eta)} b^{\varepsilon/(\varepsilon+\eta)}$. The parameter a is increased by 1%, b decreased by 1%, so price changes by $\frac{\Delta p^*}{p^*} = \left(\frac{1.01}{1.01}\right)^{1/(\varepsilon+\eta)} - 1 = 0$ and $\frac{dq^*}{q^*} = 1.01^{\eta/(\varepsilon+\eta)} 1.01^{\varepsilon/(\varepsilon+\eta)} - 1 = 1\%$.

2. Let T be the number of trusses. The number of cabinets is C where $C = 5\sqrt{400 - M} = 5\sqrt{400 - T^2}$. Thus the PPF is an ellipse,



3. Profits in thousands are $4T + C = 4T + 5\sqrt{400 - T^2}$. The first order condition for a maximum is $4 - \frac{5T}{\sqrt{400 - T^2}} = 0$, implying $4\sqrt{400 - T^2} = 5T$, or $16(400 - T^2) = 25T^2$, or $T = \sqrt{\frac{6400}{41}} = 12.49$.

4. Let T be tinting. If T tints are done, there are 400-2T hours to spend on alarms, producing $\sqrt{400 - 2T}$ alarm installations.



5. The movement of jobs represents gains from trade – overall as a country we buy telephone services or car parts for less than our costs of providing them, creating gains from trade. The production of car parts requires a medium skill level, e.g. high school graduate with some training. The subcontracted telephone services are often for a somewhat higher skill level, and the individuals who are hurt are those that would be hired if the foreign production were not available.

6. Comparative advantage is a lower cost of producing one item, expressed in terms of another item. Thus a comparative advantage in producing wheat means one can produce wheat per cell phone.

Generally, one would expect that dissimilar nations have larger gains from trade – dissimilar nations have different technologies and inputs and that means there are larger differences in opportunity costs. As a practical matter, there is more trade among developed nations than there is between developed and less developed nations.

Ec 11 Homework 3
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CALTECH

For questions 1-6, Jonah's turnip farm uses two kinds of fertilizer, organomulch and miraclepost. After some experimentation, Jonah finds that the quantity (in kg) of turnips produced per square meter of land is

$$T = 100x^{0.4}y^{0.2}$$

where x is the kilograms of organomulch and y the kilograms of miraclepost used per square meter.

1. If organomulch costs $10/kg, and miraclepost costs $20/kg, what is the relative proportion of miraclepost to organomulch that Jonah should use to maximize profit?
2. Suppose Jonah can sell the turnips for $1/kg. What quantity of fertilizers should he use to maximize profit, how many turnips does he produce, and what is the profit (per square meter of land)?
3. If the price of turnips rises to $4/kg, what happens to Jonah's profit-maximizing quantity supplied?
4. Plot Jonah's supply of turnips (in the usual way, with price on the vertical axis and quantity on the horizontal axis) as the price ranges between $0/kg and $4/kg.
5. Does Jonah have an economy of scale or a diseconomy of scale?
6. Suppose Jonah can grow tomatoes on the same land and earn $160 per square meter. At what price does Jonah quit growing turnips?
7. *Cockadoodles* are a mixture of cocker spaniels and poodles, created to make a non-shedding cocker spaniel. Cockadoodles sell for about $1500, while pure bred poodles and cocker spaniels sell for $500 to $600. Are there constant returns to scale in the production of cockadoodles? Why or why not? Assuming costs of breeding and raising them are similar to the other breeds, what is the (approximate) long-run price of cockadoodles? What limits short-run supply?

1. One way to solve this problem is to minimize the cost subject to a given production level. To do this, solve for $y = \left(\frac{T}{100x^{.4}}\right)^5 = \frac{T^5}{10^{10}x^2}$. Thus the cost of producing T is

$C = 10x + 20y = 10x + \frac{20T^5}{10^{10}x^2}$. The cost is minimized when

$0 = 10 - \frac{40T^5}{10^{10}x^3}$, or $x = \sqrt[3]{\frac{4T^5}{10^{10}}}$. Thus, $\frac{x}{y} = \frac{x}{\frac{T^5}{10^{10}x^2}} = \frac{10^{10}x^3}{T^5} = \frac{10^{10}}{T^5}\frac{4T^5}{10^{10}} = 4.$

2. Jonah wants to maximize

$$T - 10x - 20y = T - 10x - \frac{20T^5}{10^{10}x^2} = T - 10\sqrt[3]{\frac{4T^5}{10^{10}}} - \frac{20T^5}{10^{10}}\left(\frac{4T^5}{10^{10}}\right)^{-2/3}$$

$$= T - 10\sqrt[3]{\frac{4T^5}{10^{10}}} - 5\sqrt[3]{\frac{4T^5}{10^{10}}} = T - 15\left(\frac{4T^5}{10^{10}}\right)^{1/3}$$

The first order condition satisfies

$$0 = 1 - 5\left(\frac{4T^5}{10^{10}}\right)^{-2/3}\frac{20T^4}{10^{10}}$$

$$\left(\frac{4T^5}{10^{10}}\right)^{2/3} = \frac{100T^4}{10^{10}}$$

$$\left(\frac{4T^5}{10^{10}}\right)^2 = \left(\frac{100T^4}{10^{10}}\right)^3$$

$$\frac{16T^{10}}{10^{20}} = \frac{T^{12}}{10^{24}}$$

$$16 \times 10^4 = T^2$$

T=400.

Routine calculations show x =16, y =4 and profits are 160.

3. A repeat of the previous calculation shows (or better still, use the answer below)

T = 3200, x=512, y=128 and profits are 5120.

4. Jonah wants to maximize

$$pT - 10x - 20y = pT - 10x - \frac{20T^5}{10^{10}x^2} = pT - 10\sqrt[3]{\frac{4T^5}{10^{10}}} - \frac{20T^5}{10^{10}}\left(\frac{4T^5}{10^{10}}\right)^{-2/3}$$

$$= pT - 10\sqrt[3]{\frac{4T^5}{10^{10}}} - 5\sqrt[3]{\frac{4T^5}{10^{10}}} = pT - 15\left(\frac{4T^5}{10^{10}}\right)^{1/3}$$

The first order condition satisfies

$$0 = p - 5\left(\frac{4T^5}{10^{10}}\right)^{-2/3}\frac{20T^4}{10^{10}}$$

$$p\left(\frac{4T^5}{10^{10}}\right)^{2/3} = \frac{100T^4}{10^{10}}$$

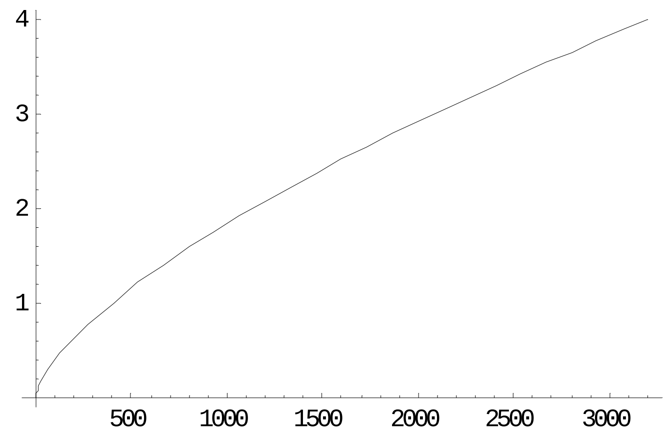
$$p^3\left(\frac{4T^5}{10^{10}}\right)^2 = \left(\frac{100T^4}{10^{10}}\right)^3$$

$$p^3\frac{16T^{10}}{10^{20}} = \frac{T^{12}}{10^{24}}$$

$$16\times10^4 p^3 = T^2$$

$$T = 400p^{3/2}.$$

$$\left(\frac{T}{400}\right)^{2/3} = p$$



5. Diseconomy of scale. If he doubles his inputs, output rises by $2^{0.8} < 2$.

6. $1. He earned $160/sq m at a price of $1.

7. There are approximately constant returns to scale in cockadoodles; the main inputs are land, food, breeding stock, veterinary care, and human supervision, and the costs of these inputs are approximately constant. None of these inputs is likely to become constrained in the near term, except breeding stock, which can only be expanded slowly. This is why prices are high now, but should fall to those commensurate with other breeds over time.

Ec 11 Homework 4
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1. Consider a person that owes $1000 on a credit card that charges 1.5% per month in interest. If the person can consolidate that $1000 debt into their mortgage at 0.5% monthly interest rate, what is the present value of the savings over ten years?
2. You are buying a $2000 big screen TV at best buy. The most you can spend is $100 per month to pay it off. You can buy it with your regular Visa and pay 1% per month in interest. How long does it take you to pay off the TV?
3. Best buy offers a "No interest for a year" deal on the TV in the previous question. However, the fine print says that, if you haven't *fully* paid for the TV in twelve months, you must pay *all* the interest. Their interest rate is 19.8% annual. How many months does it take to pay off the TV at $100 month? [Note: Most "no interest for a year" deals have this feature; Home Depot's interest rate is 21% annual, Sears' nearly 24.9% annual.]
4. Exercise 4.3.2.1 (non-university earnings should read $25,000 *per year*)
5. Exercise 4.3.2.2
6. Exercise 4.3.4.1

1. The gain is the present value of 1% per month interest on $1000 over ten years. This is a savings $10 per month, for 120 months. But what is the present value? At 0.5% per month, it is

$$PV = \frac{\$10}{r}\left(1-\frac{1}{(1+r)^n}\right) = \frac{\$10}{.005}\left(1-\frac{1}{(1.005)^{120}}\right) = \$900.74$$

At 1.5% per month, it is

$$PV = \frac{\$10}{r}\left(1-\frac{1}{(1+r)^n}\right) = \frac{\$10}{.015}\left(1-\frac{1}{(1.015)^{120}}\right) = \$554.99$$

Since the consumer is willing to borrow at ½% per month, and is avoiding 1.5% per month, the exact answer should be somewhere in between these two figures.

2. We are solving

$$\$2000 = PV = \frac{\$100}{r}\left(1-\frac{1}{(1+r)^n}\right) = \frac{100}{.01}\left(1-\frac{1}{(1.01)^n}\right)$$

or $.2 = \left(1-\frac{1}{(1.01)^n}\right)$ or $.8 = \frac{1}{(1.01)^n}$ or $Log(.8) = -nLog(1.01)$ or

$$n = \frac{-Log(.8)}{Log(1.01)} = 22.43 \text{ months}$$

3. First, note that you wind up with the 19.8% interest rate, which means you are paying a bit more than 1.517% per month. Since you are paying $100 per month, the solution is identical to the previous problem but with the 1.5% interest rate, so

$$\$2000 = \frac{100}{.01517}\left(1-\frac{1}{(1.01517)^n}\right)$$

which solves for just slightly more than 24 months.

4. The calculation is a loss of $45K per year for four years, followed by 40 years of a 25K gain. The PV of the loss is

$$PV = \frac{-\$45}{.07}\left(1-\frac{1}{(1.07)^4}\right) = -152{,}425.$$

The PV of the gain is

$$PV = \frac{1}{(1.07)^4}\frac{\$25}{.07}\left(1 - \frac{1}{(1.07)^{40}}\right) = \$254{,}267$$

The net gain is $101,842.

5. The difference is a loss of $19,000 for 4 years, followed by a gain of $10,000 for 40 years. Using the previous logic, the PV of the loss is

$$PV = \frac{-\$19}{.07}\left(1 - \frac{1}{(1.07)^4}\right) = -64{,}357.$$

The PV of the gain is

$$PV = \frac{1}{(1.07)^4}\frac{\$10}{.07}\left(1 - \frac{1}{(1.07)^{40}}\right) = \$101{,}707$$

The private school is the better deal.

6. The proportion of the resources used (from the text) is $1-(1+r)^{-\varepsilon}$, which is a whopping 17%! As the interest rate rises, the proportion used annually rises.

Ec 11 Homework 5
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Winter 2005

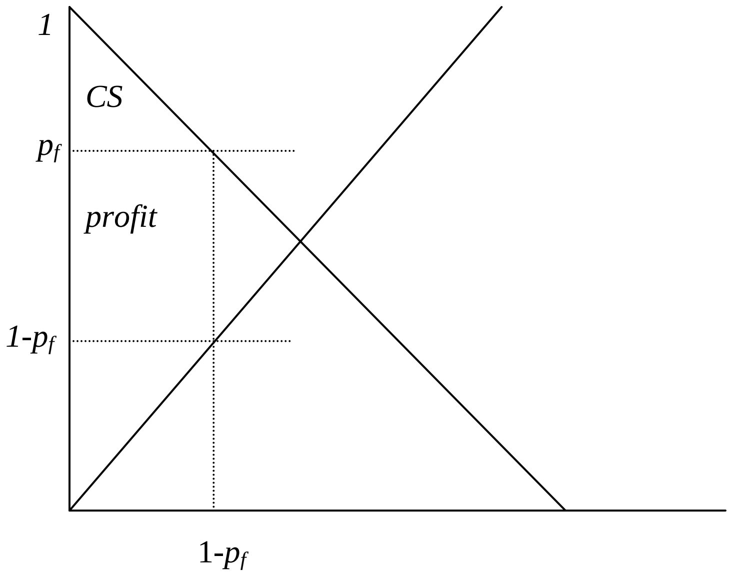
CALTECH

Suppose the quantity demanded $q_d = 1 - p$ and the quantity supplied is p, for price p between zero and one.

1. Compute the equilibrium price and quantity, the consumer surplus, and the producer surplus.
2. Suppose a price floor $p_f \geq ½$ is imposed. Compute the (standard) dead weight loss, the consumer surplus and the producer surplus. For what prices do producers benefit?
3. Now suppose producers must "produce in advance." That is, they produce the good, and then sell with the same probability as any other seller. If q units are produced, the highest cost seller has cost q, and sells with probability $\frac{1-p_f}{q}$. This seller's profit, then, is $p_f \frac{1-p_f}{q} - q = 0$, which solves for the equilibrium q. Compute the total cost (since all sellers up to q) produce.
4. Compute the producer surplus, which is revenues $p_f\,(1 - p_f\,)$ minus your part 3 answer. Do producers on average gain from an increase in the price floor above the equilibrium price?
5. Now suppose the sellers can wait until they find a buyer before committing to producing. What is the marginal producer? What is the total cost of production? What are average producer profits?
6. Exercise 6.3.1.1
7. Explain the advantage of tradable pollution permits over a historical quota.
8. Consider the theory in section 6.3.6 on fishing and extinction. Discuss two different ways of solving the tragedy of the commons, and their advantages and disadvantages. (One paragraph is sufficient.)
9. Why does voting on tax rates do better than voluntary contributions in producing a public good?
10. An addict suffers a loss of 20 from not consuming drugs, and a gain of *30 from consuming them. What is the most the addict will pay to get the drugs?

1. $1 - p^* = q_d = q_s = p^*$ so $p^* = ½$, $q^* = ½$. CS = profit = 1/8.

2. CS = ½ $(1-p_f)^2$ and profit is $(2p_f - 1)(1- p_f) + ½ (1-p_f)^2 = ½(1- p_f)(3p_f - 1)$



Profit exceeds 1/8 if and only if $4(1- p_f)(3p_f - 1) > 1$,

if and only if $-12 p_f^2 + 16 p_f - 5 > 0$,

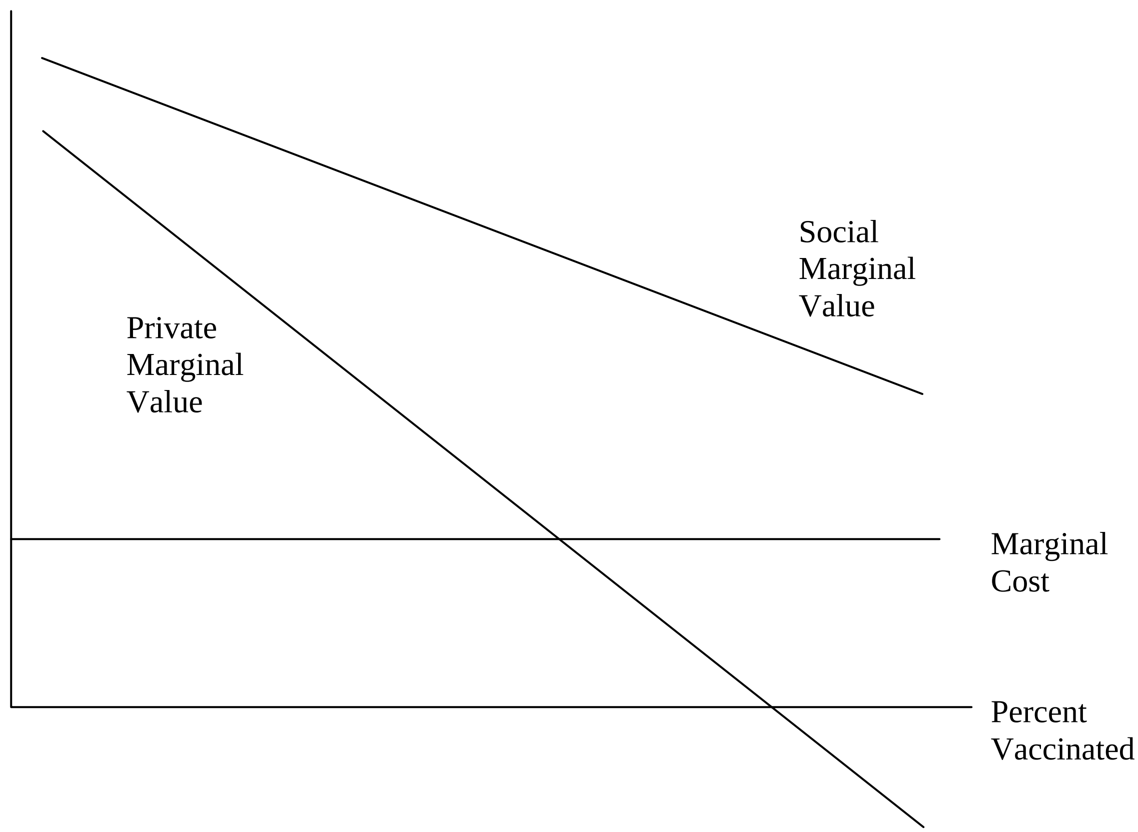
if and only if $½ < p_f < 5/6$.

3. $q = \sqrt{p_f (1 - p_f)}$ and the cost of producing q units is $½ q^2 = ½ p_f (1 - p_f)$.

4. Revenues are $p_f (1 - p_f)$ so profits are revenues minus total costs, which gives profits of $½ p_f (1 - p_f)$. This is maximized at the competitive price p_f – the producers do not benefit from a price ceiling.

5. The marginal producer – since production takes place only when a buyer is found – is the producer with marginal cost p_f. Thus the average cost of production is ½ p_f (the average of all producers less than p_f) and the total cost is just the average cost times the quantity $1 - p_f$, giving the same answer as the previous question.

6. The private benefit could easily be negative provided there is a positive social benefit. This is in fact thought by many to be true about polio vaccinations.



7. The main advantages are (i) efficient allocation of pollution, (ii) pricing the actual pollution, so that the benefits of pollution are known. A historical quota might reflect value initially, but as the world changes, the historical quota gets more and more inefficient.

8. (1) Sell “fishing rights.” (2) Tax fish. (3) Auction fishing rights with a fixed quota. Any of these theoretically can limit the overuse to efficient levels. Taxes and selling rights let the quantity adjust to changing circumstances, which can require adjusting the taxes as demand changes. Unless the taxes can be readily adjusted, the same problem can arise if the tax is too small. Quotas don’t respond to demand, so that fishing can be too large in years when demand is small. Quotas may also need adjustment, although less so than pricing the use of the resource.

9. Voluntary contributions tend to be the unilateral contribution of a single individual with the highest value, while voting on taxes creates the median value times the total number, and works perfectly when everyone has the same preferences for the public good. Basically, in a population of *n* individuals, *n* times the median usually exceeds the maximum.

10. An addict suffers a loss of 20 from not consuming drugs, and a gain of *30 from consuming them. What is the most the addict will pay to get the drugs?

50 – the net gain from purchasing.

CALTECH

Ec 11 Homework 6
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Winter 2005

7. A monopolist faced demand $q = 1 - p$. The monopolists costs are $c(q) = .125 + .25q$.
    a. What is the monopoly price and quantity? What are monopoly profits?
    b. What is the efficient (maximizes gains from trade) price and quantity? What are profits are this price?
    c. If the monopoly has to at least break even, what is the socially optimal *price cap*?
8. An *ad valorem* tax is a proportional tax on value, like a sales tax of 8%. Some excise taxes (taxes on specific items) are a constant amount, like a sixteen cent per gallon gasoline excise tax. Suppose a monopolist currently sells one million units at a price of $1 each (this is the pre-tax monopoly price), and consider a small tax. Does a small ad valorem tax reduce output more, or less, than the same per unit tax? [Hint: consider the monopoly's maximization problem for each kind of tax and then express the first order condition as marginal revenue equal to an adjusted cost function.] [For 50% credit you can solve this problem for the special case of linear demand and constant marginal cost.]
9. Cable TV is generally a local monopoly because it is expensive to have two sets of wires to create competition. A community could capture the monopoly profits by selling the right to operate the cable monopoly, and then redistributing the proceeds to the community. Would the outcome of this process be efficient? Why or why not?
10. Identify whether the following are direct price discrimination, indirect price discrimination, or not price discrimination, and why.
    a. senior citizen's discount at the movies
    b. "Buy 9, get one free" at Chuck's Coffee
    c. "Shoe, buy one and get one free"
    d. Discount to Disney on the side of Coca-cola can
    e. Matinee discount at the movies
    f. Discounted price for Windows with the purchase of a new PC
    g. Sam's Club discount on 48 rolls of paper towels
11. Should Apple discount the price of ipods sold in retirement communities? Why or why not?
12. In Akerlof's market for lemons model, suppose it is possible to certify cars, verifying that they are better than a particular quality q^*. Thus, a market for cars "at least as good as q^*" is possible. What price or prices are possible in this market? [Hint: sellers offer cars only if $q^* \leq$ quality $\leq p$.] Thought question: What quality maximizes the expected gains from trade?

1. a.The monopolist earns $pq - c(q) = q(1 - q) - .125 - .25q$. The first order condition gives

$$1 - 2q - .25 = 0,$$

or $q = .375$

The monopoly price is 1-q=.625 and the profits are .015625.

b. The efficient price sets marginal value 1-q equal to marginal cost .25, so q = .75 and the price is .25. Profits at this price are -0.125.

c. The lowest price (highest quantity) giving the monopolist non-negative profits satisfies

$$0 = pq - c(q) = q(1 - q) - .125 - .25q$$

which solves for q={¼, ½ } and the relevant root is q = ½ , with a price of ½ .

2. Let q_0 be the pretax monopoly quantity. In one case, the monopolist earns

$$(p(q) - tp(q_0))q - c(q)$$

where t is the per unit tax rate. The first order condition is

(1) $$0 = p'(q)q + p(q) - c'(q) - tp(q_0)$$

In the other case, the monopolist earns

$$(1 - t)p(q)q - c(q)$$

with associated first order condition

(2) $$0 = (1 - t)(p'(q)q + p(q)) - c'(q)$$

Let q_1 be the solution to (1) and q_2 be the solution to (2). Then

$$p'(q_1)q_1 + p(q_1) = c'(q_1) + tp(q_0)$$

and

$$p'(q_2)q_2 + p(q_2) = \frac{c'(q)}{1 - t}$$

For t near zero, $tp(q_0) - \frac{c'(q)}{1-t} > 0$.

Since marginal revenue is decreasing and price exceeds marginal cost, q_1 is smaller than q_2 and thus charging a proportional tax reduces the monopoly quantity less.

3. The outcome is inefficient because the monopoly sets the price higher than marginal cost, and thus too few units are purchased.

4.

senior citizen's discount at the movies are direct price discrimination
"Buy 9, get one free" at Chuck's Coffee, quantity discount, so yes
"Shoe, buy one and get one free", generally not; since there are few consumers who would like to buy them separately.
Discount to Disney on the side of Coca-cola can – like a coupon, so yes
Matinee discount at the movies – no, since product is different (different time)
Discounted price for Windows with the purchase of a new PC – yes, since based on other purchases
Sam's Club discount on 48 rolls of paper towels, yes, quantity discount

5. In favor: lower willingness to pay generally along with very high markups. Problem is arbitrage – would the discount to senior citizens become general? One solution is to mail rebates only to addresses in the retirement communities, which makes arbitrage harder.

6. sellers offer cars only if $q^* \leq$ quality $\leq p$. This gives an average quality of ½ (p+q^*). Since buyers must be willing to buy, we have that price is no larger than the 3/2 times the average quality, or ¾ (p+q^*), which gives $p \leq$ ¾ (p+q^*), or $p \leq 3q^*$. Thus, the possible prices is the interval [q^* , $3q^*$].

Which one maximizes the gains from trade? The gains from trade is just ½ times the total quality traded, which is proportional to p-q^*. This is maximized at $p = 3q^*$ and then $q^* = 1/3$.

Ec 11 Homework 7
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CALTECH

13. Exercise 7.1.3.2
14. Exercise 7.1.3.3
15. Exercise 7.1.4.4
16. Exercise 7.1.4.5
17. Exercise 7.1.6.1

1. Show that, in the Paper, Scissors, Rock game, there are no pure strategy equilibria. Show that playing all three actions with equal likelihood is a mixed strategy equilibrium.

Fixing one player's strategy, the other player can always get 1. Thus, for any pair of pure strategies, both players must get 1, which is impossible.

If player 1 plays all three actions with equal probability, then player 2 gets zero no matter what strategy player 2 follows. Thus, any strategy maximizes expected profit for player 2, and in particular, playing all three actions with equal probability maximizes player 2's expected profit. The game is symmetric, so the same applies to player 1.

2. Find all equilibria of the following games:

1 Column

		Left	Right
Row	Up	(3,2)	(11,1)
	Down	(4,5)	(8,0)

Column has a dominant strategy of left, so (Down, left) is the unique equilibrium.

2 Column

		Left	Right
Row	Up	(3,3)	(0,0)
	Down	(4,5)	(8,0)

Column has a dominant strategy of left, so (Down, left) is the unique equilibrium.

3 Column

		Left	Right
Row	Up	(0,3)	(3,0)
	Down	(4,0)	(0,4)

No pure strategy equilibria. Row plays up with prob p satisfying $3p = 4(1-p)$, so $p = 4/7$. Similarly Column plays left with probability q satisfying $4q=3(1-q)$, so $q = 3/7$.

4 Column

		Left	Right
Row	Up	(7,2)	(0,9)
	Down	(8,7)	(8,8)

Row has a dominant strategy to play down, so column plays right, and (down, right) is the unique equilibrium

5

		Column	
		Left	Right
Row	Up	(1,1)	(2,4)
	Down	(4,1)	(3,2)

Column plays right, so row plays down.

6

		Column	
		Left	Right
Row	Up	(4,2)	(2,3)
	Down	(3,8)	(1,5)

Row plays up, so column plays right.

3.

Table : Avoiding Rocky

		Rocky	
		Party 1	Party 2
You	Party 1	(5,15)	(20,10)
	Party 2	(15,5)	(0,20)

(i) Show there are no pure strategy Nash equilibria in this game. (ii) Find the mixed strategy Nash equilibria. (iii) Show that the probability you encounter Rocky is $\frac{7}{12}$.

First note that if you play P1, Rocky plays P1, in which case you prefer P2, in which case Rocky prefers P2, in which case you prefer P1. So no pure strategy equilibrium exists.

If you go to P1 with probability p, Rocky is indifferent if $15p + 5(1-p) = 10p + 20(1-p)$, or $p = \frac{3}{4}$. If Rocky goes to P1 with probability q, you are indifferent if $5q + 20(1-q)=15q$, or $q = 2/3$. Thus, you encounter Rocky at P1 with probability $pq = \frac{1}{2}$, and at party P2 with probability $(1-p)(1-q)=1/12$, for a total of 7/12.

4.

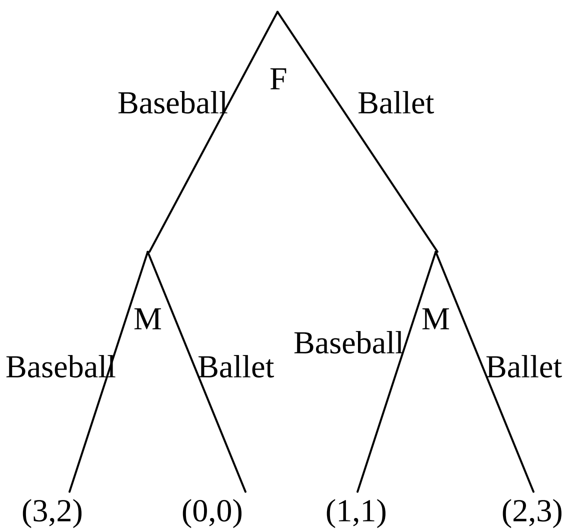
Table Price Cutting Game

		Firm 2	
		High	Low
Firm 1	High	(15,15)	(0,25)
	Low	(25,0)	(5,5)

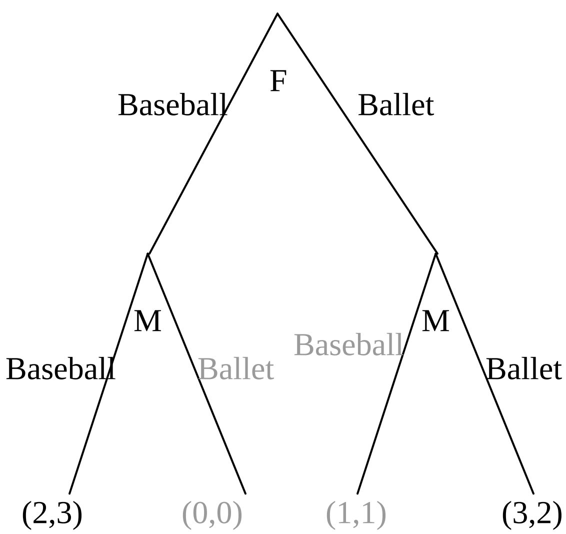
Show that the firms have a dominant strategy to price low, so that the only Nash equilibrium is (Low, Low).

If Firm 2 prices high, firm 1 gets 15 from high, 25 from low and prefers low. Similarly, if firm 2 prices low, firm 1 gets 0 from high, 5 from low and prefers low.

5. Formulate the battle of the sexes as a sequential game, letting the woman choose first. (This situation could arise if the woman can leave a message for the man about where she has gone.) Show that there is only one subgame perfect equilibrium, and that the woman enjoys a first-mover advantage over the man, and she gets her most preferred outcome.



Using grey to indicate the choices the man won't make,



This leaves the woman with a payoff of 3 from Ballet, 2 from Baseball, so she chooses ballet with a payoff of 3.

CALTECH

Ec 11 Homework 8
Professor R. Preston McAfee
Winter 2005

18. Consider two stores, which are only considering two markups over marginal cost, $1 and $2. 10% of the customers go to store 1 and buy at either price. 10% go to store 2 and buy at either price. Finally, 80% go to the cheaper store (splitting equally in the event of a tie). Formulate this situation as a simultaneous move game played by the stores and compute the equilibrium price dispersion.
19. Suppose there are *n* identical firms, playing the Cournot game, with constant marginal cost and linear demand.
    a. Solve for the equilibrium price and market quantity.
    b. What is the dead-weight loss of imperfect competition?
20. “Imperfectly competitive markets do too much R&D and waste resources advertising.” Discuss this statement using imperfect competition theory.
21. From an agency perspective, what is the value of monitoring employees? Can employees benefit from being monitored?

1. Let p represent the probability of a price equal to 2. The payoff to a firm, given the other firm charges 2 with probability p and 2 with probability 1-p, from price 2 is:

$\pi_2 = 2 \times (.1 + p \,½\, (.8)\,) = .2\,(1+ 4p)$

This comes about because quantity is 25% loyal customers, plus equally splitting 50% when the other firm charges 2, with probability p.

If the firm charges a markup of 1, it obtains

$\pi_2 = .1 + (1\text{-}p)\,½\,(.8) + p\,(.8) = .1\,(1 + 4(1\text{-}p) + 8p) = .1\,(5 + 4p)$

since the firm gets its loyal customers (.1), splits the searchers when the other firm prices at 1 (probability 1-p) and gets the entire set of searchers when the other firm charges 2 (probability p).

These must be equal to obtain a mixed strategy, so 2(1+4p) = 5 + 4p, or 3 = 4p, or p = ¾.

2. Let the demand satisfy $p(Q) = a – bQ$, where Q is industry output, and marginal cost be c. Hypothesize that each firm produces q^*. Then a given firm's profits, as a function of its quantity q, are

$$(a - b((n-1)q^* + q))q - cq\,.$$

This is maximized at a value of q satisfying

$$a - b((n-1)q^* + 2q) - c = 0$$

A symmetric equilibrium has each firm choose $q=q^*$, which entails

$$a - b(n+1)q^* - c = 0$$

or $q^* = \dfrac{a-c}{b(n+1)}$

Industry output is nq^* and the equilibrium price is $a - bnq^* = \dfrac{a+nc}{n+1}$. This yields a dead weight loss of

$$½\,(p - c)(q_c - nq^*) = ½\left(\frac{a+nc}{n+1} - c\right)\left(\frac{a-c}{b} - \frac{n(a-c)}{b(n+1)}\right) = \frac{(a-c)^2}{2b(n+1)^2}\,.$$

3. The Hotelling circle model produces too many firms. Can this be used to justify the assertion that firms engage in too much R&D or advertising? Certainly not directly, since additional R&D from a given firm is not equivalent to additional firms.

However, imperfect competition models yield, as an equilibrium outcome, positive profits on each customer. Such profits provide an incentive to increase market share, through means other than price, like advertising and R&D. Will this result in “too much” advertising and R&D? Such efforts are wasteful in one sense, because their purpose is to move customers from one seller to another. These efforts substitute for lower price, however, which tends to return more gains from trade to consumers and mitigate the dead weight loss, which is socially beneficial. Depending on the details of the model, prohibiting advertising might either increase or decrease the overall gains from trade.

4. Monitoring is a substitute for incentive contracts. Since there is a social loss associated with the inability to observe the agents’ action, a loss that both reduces the agent’s effort below that which would prevail under full information and also imposes risk on the agent. By increasing the gains from trade between employer and employee, monitoring may improve the payoffs for both parties.

In the textbook agency model, the agent is pushed up against the willingness to work, and thus agents don’t benefit from monitoring.