Game Plan

- Game theory review
- Review of some basics
- Questions before the final

Answering GT Questions

- Equilibrium is a set of strategies, not payoffs
  - Ex: The Nash equilibrium is (Up, Left), NOT (4, 3)
  - Mixed equilibrium example: “Row plays Up with probability 3/4 and Down with probability 1/4. Column plays Left with probability 1/2 and Right with probability 1/2.”

Answering GT Questions

- Nash equilibria can be either pure or mixed strategy equilibria
Why do you need to make other person indifferent?

- “Odds and Evens”
- Two people: “Odds” and “Evens”
- Each hold out 1 or 2 fingers, if total number is odd, Evens gives Odds a dollar, otherwise Odds gives Evens a dollar

<table>
<thead>
<tr>
<th></th>
<th>1 finger</th>
<th>2 fingers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odds</td>
<td>-1, 1</td>
<td>1, -1</td>
</tr>
<tr>
<td>Evens</td>
<td>1, -1</td>
<td>-1, 1</td>
</tr>
</tbody>
</table>

Why do you need to make other person indifferent?

- Obviously, mixing is necessary
- Let’s say “Evens” plays 1 finger 3/4 of the time, 2 fingers 1/4 of the time

Odds makes $1

Evens’ disembodied hand

Odds’ disembodied hand

Why do you need to make other person indifferent?

- Expected value to Odds of playing 2 fingers: \(\frac{3}{4} (1) + \frac{1}{4} (-1) = \frac{1}{2}\)
- Expected value to Odds of playing 1 finger: \(\frac{3}{4} (-1) + \frac{1}{4} (1) = -\frac{1}{2}\)

Odds loses $1

Odds’ olde-timey hand
**Why do you need to make other person indifferent?**

- Expected value to Odds of playing 2 fingers: \[
\frac{3}{4} \cdot 1 + \frac{1}{4} \cdot (-1) = \frac{1}{2}
\]
- Expected value to Odds of playing 1 finger: \[
\frac{3}{4} \cdot (-1) + \frac{1}{4} \cdot 1 = -\frac{1}{2}
\]
- So Odds would always play 2 fingers
- But then, Evens would always want to play 2, and we know this isn’t an equilibrium

**Another way to think of it**

- Tennis: Server deciding to serve to the forehand or backhand
- Probability that receiver returns serve:

<table>
<thead>
<tr>
<th>Server’s Aim</th>
<th>Forehand</th>
<th>Backhand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forehand Move</td>
<td>90%</td>
<td>20%</td>
</tr>
<tr>
<td>Backhand</td>
<td>30%</td>
<td>60%</td>
</tr>
</tbody>
</table>

**Another way to think of it**

Percentage of successful returns

- Anticipate backhand
- Anticipate forehand

Percentage of times server aims to forehand
Another way to think of it

- Examples taken from *Thinking Strategically* by Avinash K. Dixit and Barry J. Nalebuff

Tips for simultaneous move games

- If you find only one pure strategy equilibrium, there shouldn’t be a mixed eq.
- If you find two pure strategy equilibria (like in Battle of the Sexes), look for a mixed one
- In a given square, ask if anyone could do better by moving if the other guy stays

<table>
<thead>
<tr>
<th></th>
<th>Baseball</th>
<th>Ballet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseball</td>
<td>3, 2</td>
<td>1, 1</td>
</tr>
<tr>
<td>Ballet</td>
<td>0, 0</td>
<td>2, 3</td>
</tr>
</tbody>
</table>
Sequential games

- Work your way backwards!

Comparison of Industry Types

<table>
<thead>
<tr>
<th></th>
<th>Perfect Competition</th>
<th>Monopoly</th>
<th>Cournot Oligopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Firms</td>
<td>Large</td>
<td>1</td>
<td>Small</td>
</tr>
<tr>
<td>Price</td>
<td>Given</td>
<td>Function of q</td>
<td>Function of Q (industry total)</td>
</tr>
<tr>
<td>How quantity is chosen</td>
<td>Choose q s.t. p = MC</td>
<td>Choose q s.t. MR = MC</td>
<td>Max profit, given what other firm(s) does (do)</td>
</tr>
<tr>
<td>Profit function</td>
<td>p_i - c_i(q_i)</td>
<td>p(q) - c(q)</td>
<td>p(q_i)q_i - c(q)</td>
</tr>
<tr>
<td>Profit level</td>
<td>0 in long run</td>
<td>High</td>
<td>Between PC and monopoly</td>
</tr>
<tr>
<td>Dead weight loss</td>
<td>None</td>
<td>Triangle from q_m to q_e below p_m and above MR at q_e</td>
<td>Triangle between Q_e and Q_m and p_e and p_m plus production inefficiency</td>
</tr>
</tbody>
</table>

Monopoly Deadweight Loss

[Diagram of supply and demand curves showing deadweight loss]
Cournot Example

- 2 firms, with marginal costs $MC_1 = 3$, $MC_2 = 2$
- $p(Q) = 5 - 2Q$ (Q is total quantity)
- Want to find $q_1$ and $q_2$, total quantity, and price
- Steps:
  - Find FOC's for firms 1 and 2
  - Solve for $q_1$ in terms of $q_2$ and vice versa
  - Plug one quantity into the expression for the other

Cournot Example

- What is Deadweight Loss?
- Efficient solution occurs where price = lowest marginal cost
- Two components:
  - Lost gains from trade
  - Excess cost paid for quantity of goods made by higher cost producer
Remember!
- Read questions carefully
- Make sure you answer all parts of a question
- Show us your work, explain thought process

Sequential Battle of the Sexes

<table>
<thead>
<tr>
<th></th>
<th>Baseball</th>
<th>Movie</th>
</tr>
</thead>
<tbody>
<tr>
<td>man</td>
<td>3, 2</td>
<td>1, 1</td>
</tr>
<tr>
<td>man</td>
<td>0, 0</td>
<td>2, 3</td>
</tr>
</tbody>
</table>

Monopoly problem (last year’s final)
- Consider a monopolist with constant marginal cost facing linear demand. A unit tax of $t$ is imposed on the monopolist. By how much does the price rise?
- Linear demand: $q(p) = a - bp$
- Let marginal cost be “c”
- Without tax, profits are $(p-c)q(p) = (p-c)(a-bp)$
Monopoly problem (last year’s final)

\[
\pi = (p-c)(a-bp)
\]

\[
d\pi/dp = a-2bp+cb = 0
\]

\[
2bp = a+cb
\]

\[
p = \frac{1}{2} (c + a/b)
\]

Tax is part of marginal cost, so imagine
\[
c = c + t.
\]
Then price rises by 1/2 \(t\).
Could also write out \(\pi = (p-c-t)(a-bp)\) and solve as above.