Perfect Competition

Major Points
• Focus on firm behavior
• Choices when prices are exogenous
• Profit maximization constrained by technology
  – calculate input demands
  – comparative statics
  – conclusions about individual firm behavior
• Aggregate to market
  – market dynamics

Types of Firms
• Proprietorship, e.g. family business
• Partnership, e.g. law, accounting practice
• Corporation
  – limited liability by shareholders
  – legal “person”
  – managed by agents of shareholders
• Non-profit corporation
  – only certain activities achieve tax-free status
Organizational Form

- Proprietorship: decisions made by owner
- Partnership: voting and negotiation
- Corporation: delegation
  - shareholders elect board
  - board chooses management
  - management makes most decisions
  - some large decisions require board vote
  - "separation of ownership and control"

Production Functions

- Focus on a single output
- Cobb-Douglas
  \[ f(x_1, x_2, \ldots, x_n) = a_0 x_1^{a_1} x_2^{a_2} \ldots x_n^{a_n} \]
- Fixed proportions
  \[ f(x_1, x_2, \ldots, x_n) = \min \{a_1 x_1, a_2 x_2, \ldots, a_n x_n \} \]
  - Perfect complements
- Perfect Substitutes arises when the components enter additively

Cobb-Douglas Isoquants

[Diagram of Cobb-Douglas Isoquants]
Marginal Product

- Marginal product of capital is $\frac{\partial f}{\partial K}(K, L)$
- Will sometimes denote $f_K = f_1 = \frac{\partial f}{\partial K}(K, L)$
- Some inputs more readily changed than others
- Suppose $L$ changed in short-run, $K$ in long-run

Complements and Substitutes

- Increasing amount of a complement increases productivity of another input:
  $\frac{\partial^2 f}{\partial K \partial L} > 0$
- Substitutes
  $\frac{\partial^2 f}{\partial K \partial L} < 0$

Short Run Profit Maximization

$\pi = pF(K, L) - rK - wL$.

0 = \frac{\partial \pi}{\partial L} = p \frac{\partial F}{\partial L}(K, L^*) - w. \quad \text{FOC}

0 \geq \frac{\partial^2 \pi}{(\partial L)^2} = p \frac{\partial^2 F}{(\partial L)^2}(K, L^*). \quad \text{SOC}
Graphical Depiction

Short-run Effect of a Wage Increase

\[ 0 = p \cdot \frac{\partial^2 F}{(\partial L)^2} (K, L^* (w)) L^{**} (w) - 1, \]
\[ L^{**} (w) = \frac{1}{p \cdot \frac{\partial^2 F}{(\partial L)^2} (K, L^* (w))} \leq 0. \]

Aside: Revealed Preference

- Revealed preference is a powerful technique to prove comparative statics
- Works without assumptions about continuity or differentiability
- Suppose \( w_1 < w_2 \) are two wage levels
- The entrepreneur chooses \( L_1 \) when the wage is \( w_1 \) and \( L_2 \) when the wage is \( w_2 \)
Revealed Preference Proof

Prefer $L_1$ to $L_2$ when wage $= w_1$

$$pf(K, L_1) - rK - w_1L_1 \geq pf(K, L_2) - rK - w_1L_2$$

Prefer $L_2$ to $L_1$ when wage $= w_2$

$$pf(K, L_2) - rK - w_2L_2 \geq pf(K, L_1) - rK - w_2L_1.$$ 

Sum these two

$$pf(K, L_1) - rK - w_1L_1 + pf(K, L_2) - rK - w_2L_2 \geq$$

$$pf(K, L_1) - rK - w_2L_1 + pf(K, L_2) - rK - w_1L_2$$

Revealed Preference, Cont’d

- Cancel terms to obtain
  
  $$-w_1L_1 - w_2L_2 \geq -w_2L_1 - w_1L_2$$
  
  or
  
  $$(w_1 - w_2)(L_2 - L_1) \geq 0.$$ 

- Revealed preference shows that profit maximization implies $L$ falls as $w$ rises.

Comparative Statics

- What happens to $L$ as $K$ rises?

$$L^{**}(K) = \frac{-\partial^2 F}{\partial K \partial L} (K, L^*(K))$$

$$\frac{\partial^2 F}{(\partial L)^2} (K, L^*(K))$$

- Thus, $L$ rises if $L$ and $K$ are complements, and falls if substitutes
### Applications

- Computers use has reduced demand for secretarial services (substitutes)
- Increased technology generally has increased demand for high-skill workers (complements)
- Capital often substitutes for simple labor (tractors, water pipes) and complements skilled labor (operating machines)

### Shadow Value

- Constraints can be translated into prices
- Marginal value of relaxing a constraint is known as *shadow value*
- Marginal cost of fixed capital

\[ \frac{d\pi(K,L^*)(K)}{dK} = \frac{\partial \pi(K,L^*)}{\partial K} = \rho \frac{\partial F}{\partial K}(K,L^*) - r \]

- May be negative if too much capital

### Cost Minimization

- Profit maximization requires minimizing cost
- Cost minimization for fixed output

\[ c(y) = \text{Min } wL + rK \]

*subject to* \( f(K,L) = y \)
Cost Minimization, Continued

- Profit maximization:
- \[ \max p_y - (wL + rK) \ \text{s.t.} \ f(K,L) = y \]
- For given \( y \), this is equivalent to minimizing cost.
- Cost minimization equation:
  \[
  \frac{\partial f}{\partial L} = \frac{\partial f}{\partial K} \frac{dK}{dL} \Big|_{f(K,L)=y} = -\frac{w}{r}
  \]

Cost Min Diagram

Short-run Costs

- Short-run total cost
  - \( L \) varies, \( K \) does not
- Short-run marginal cost
  - Derivative of cost with respect to output
- Short-run average cost
  - average over output
  - infinite at zero, due to fixed costs
- Short-run average variable cost
  - average over output, omits fixed costs
Long-run costs

- Long-run average cost
  - increasing if diseconomy of scale
  - decreasing if economy of scale
- Scale economy if, for λ > 1,
  \[ f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) > \lambda f(x_1, x_2, \ldots, x_n) \]

\[ AVC(\lambda) = \frac{w_1\lambda x_1 + w_2\lambda x_2 + \ldots + w_n\lambda x_n}{\lambda f(x_1, x_2, \ldots, x_n)} = \frac{\lambda f(x_1, x_2, \ldots, x_n)}{\lambda AVC(1)} \]

Aside: Distribution of Profits with Constant Returns to Scale

\[ \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \ldots + \frac{\partial f}{\partial x_n} \bigg|_{\lambda \to 1} = \lim_{\lambda \to 1} \frac{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) - f(x_1, x_2, \ldots, x_n)}{\lambda - 1} = f(x_1, x_2, \ldots, x_n) \]

- Thus, paying inputs their marginal product uses up the output exactly under constant returns to scale.
- Permits efficient decentralization of firm using prices

Distribution of Profits with Increasing Returns to Scale

\[ \frac{\partial f}{\partial x_1} + \frac{\partial f}{\partial x_2} + \ldots + \frac{\partial f}{\partial x_n} \bigg|_{\lambda \to 1} = \lim_{\lambda \to 1} \frac{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) - f(x_1, x_2, \ldots, x_n)}{\lambda - 1} \geq f(x_1, x_2, \ldots, x_n) \]

- Paying inputs their marginal product uses is not generally feasible
- Requires centralization of operations
Firm Costs

Min AC implies MC = AC

\[ 0 = \frac{d}{dq} \frac{C(q)}{q} = \frac{C'(q)}{q} - \frac{C(q)}{q^2} \Rightarrow C'(q) = \frac{C(q)}{q} \]

Shut down

- Firm shuts down when price < average cost
- Firm shuts down in short run when price < short run average cost = min average variable cost
- Firm exits in long run when price < long run average cost = min average total cost
Firm Reaction to Price Changes

Short run supply

Long-run Equilibrium

Increase in Demand
Large Decrease in Demand

External Economy of Scale
- The size of the industry may affect individual firm costs
  - economy of scale in input supply
  - bidding up price of scarce input
- External economy of scale means LRATC is decreasing in

General Long-run Dynamics
Markets

- University Education
- Housing
- Electric cars
- Energy
- Portable music players