Perfect Competition

Major Points

- Focus on firm behavior
- Choices when prices are exogenous
- profit maximization constrained by technology
  - calculate input demands
  - comparative statics
  - conclusions about individual firm behavior
- Aggregate to market
  - market dynamics

Types of Firms

- Proprietorship, e.g. family business
- Partnership, e.g. law, accounting practice
- Corporation
  - limited liability by shareholders
  - legal "person"
  - managed by agents of shareholders
- Non-profit corporation
  - only certain activities achieve tax free status

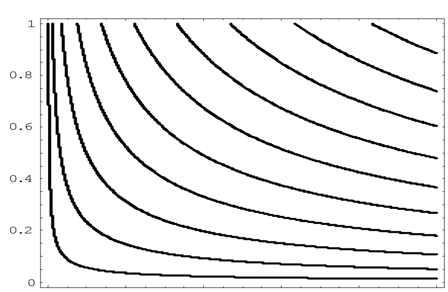
Organizational Form

- Proprietorship: decisions made by owner
- Partnership: voting and negotiation
- Corporation: delegation
  - shareholders elect board
  - board chooses management
  - management makes most decisions
  - some large decisions require board vote
  - "separation of ownership and control"

Production Functions

- Focus on a single output
- Cobb-Douglas
  $f(x_1, x_2, \ldots, x_n) = a_0 x_1^{a_1} \, x_2^{a_2} \, \ldots \, x_n^{a_n}$
- Fixed proportions
  $f(x_1, x_2, \ldots, x_n) = Min\,\{a_1 x_1, a_2 x_2, \ldots, a_n x_n\}$
  - Perfect complements
- Perfect Substitutes arises when the components enter additively

Cobb-Douglas Isoquants



Marginal Product

- Marginal product of capital is $\frac{\partial f}{\partial K}(K, L)$
- Will sometimes denote $f_K = f_1 = \frac{\partial f}{\partial K}(K,L)$
- Some inputs more readily changed than others
- Suppose L changed in short-run, K in long-run

Complements and Substitutes

- Increasing amount of a complement increases productivity of another input:

$$\frac{\partial^2 f}{\partial K \partial L} > 0$$

- Substitutes

$$\frac{\partial^2 f}{\partial K \partial L} < 0$$

Short Run Profit Maximization

$$\pi = pF(K,L) - rK - wL.$$

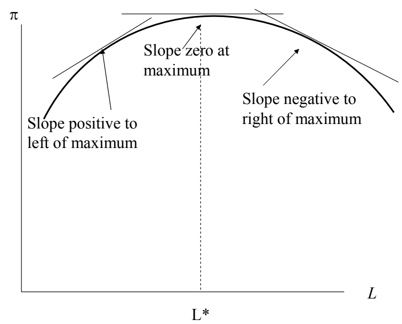
$$0 = \frac{\partial \pi}{\partial L} = p\frac{\partial F}{\partial L}(K,L^*) - w.$$

- FOC

$$0 \geq \frac{\partial^2 \pi}{(\partial L)^2} = p\frac{\partial^2 F}{(\partial L)^2}(K,L^*).$$

- SOC

Graphical Depiction



Short-run Effect of a Wage Increase

$$0 = p \frac{\partial^2 F}{(\partial L)^2}(K, L^*(w)) L^{*\prime}(w) - 1,$$

$$L^{*\prime}(w) = \frac{1}{p \frac{\partial^2 F}{(\partial L)^2}(K, L^*(w))} \le 0.$$

Aside: Revealed Preference

- Revealed preference is a powerful technique to prove comparative statics
- Works without assumptions about continuity or differentiability
- Suppose $w1 < w2$ are two wage levels
- The entrepreneur chooses $L1$ when the wage is $w1$ and $L2$ when the wage is $w2$

Revealed Preference Proof

Prefer L_1 to L_2 when wage = w_1

$$pf(K,L_1) - rK - w_1L_1 \geq pf(K,L_2) - rK - w_1L_2$$

Prefer L_2 to L_1 when wage = w_2

$$pf(K,L_2) - rK - w_2L_2 \geq pf(K,L_1) - rK - w_2L_1.$$

Sum these two

$$pf(K,L_1) - rK - w_1L_1 + pf(K,L_2) - rK - w_2L_2 \geq$$
$$pf(K,L_1) - rK - w_2L_1 + pf(K,L_2) - rK - w_1L_2$$

Revealed Preference, Cont'd

- Cancel terms to obtain

$$-w_1L_1 - w_2L_2 \geq -w_2L_1 - w_1L_2$$

or

$$(w_1 - w_2)(L_2 - L_1) \geq 0.$$

- Revealed preference shows that profit maximization implies *L* falls as *w* rises.

Comparative Statics

- What happens to *L* as *K* rises?

$$L^{*\prime}(K) = \frac{-\frac{\partial^2 F}{\partial K \partial L}(K, L^*(K))}{\frac{\partial^2 F}{(\partial L)^2}(K, L^*(K))}.$$

- Thus, *L* rises if *L* and *K* are complements, and falls if substitutes

Applications

- Computers use has reduced demand for secretarial services (substitutes)
- Increased technology generally has increased demand for high-skill workers (complements)
- Capital often substitutes for simple labor (tractors, water pipes) and complements skilled labor (operating machines)

Shadow Value

- Constraints can be translated into prices
- Marginal value of relaxing a constraint is known as *shadow value*
- Marginal cost of fixed capital

$$\frac{d\pi(K, L^*(K))}{dK} = \frac{\partial \pi(K, L^*)}{\partial K} = p\frac{\partial F}{\partial K}(K, L^*) - r$$

- May be negative if too much capital

Cost Minimization

- Profit maximization requires minimizing cost
- Cost minimization for fixed output

c(y) = Min $wL + rK$

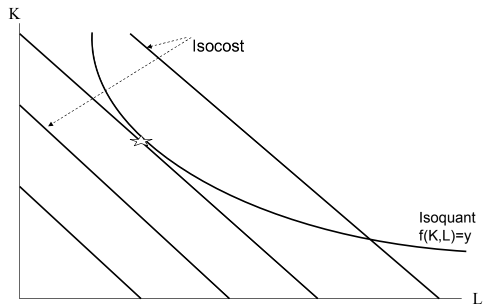
subject to $f(K, L) = y$

Cost Minimization, Continued

- Profit maximization:
- max py – (wL + rK) s.t. $f(K,L) = y$
- For given y, this is equivalent to minimizing cost.
- Cost minimization equation:

$$-\frac{\partial f/\partial L}{\partial f/\partial K} = \left.\frac{dK}{dL}\right|_{f(K,L)=y} = -\frac{w}{r}$$

Cost Min Diagram



Short-run Costs

- Short-run total cost
  - L varies, K does not
- Short-run marginal cost
  - Derivative of cost with respect to output
- Short-run average cost
  - average over output
  - infinite at zero, due to fixed costs
- Short-run average variable cost
  - average over output, omits fixed costs

Long-run costs

- Long-run average cost
  - increasing if diseconomy of scale
  - decreasing if economy of scale
- Scale economy if, for λ>1,

$$f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) > \lambda f(x_1, x_2, \ldots, x_n)$$

$$\begin{aligned}\mathrm{AVC}(\lambda) &= \frac{w_1 \lambda x_1 + w_2 \lambda x_2 + \ldots + w_n \lambda x_n}{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n)} \\ &= \frac{\lambda f(x_1, x_2, \ldots, x_n)}{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n)} AVC(1)\end{aligned}$$

Aside: Distribution of Profits with Constant Returns to Scale

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \ldots x_n \frac{\partial f}{\partial x_n} = \frac{d}{d\lambda} f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n)\Big|_{\lambda \to 1} =$$

$$= \lim_{\lambda \to 1} \frac{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) - f(x_1, x_2, \ldots, x_n)}{\lambda - 1} = f(x_1, x_2, \ldots, x_n)$$

- Thus, paying inputs their marginal product uses up the output exactly under constant returns to scale.
- Permits efficient decentralization of firm using prices

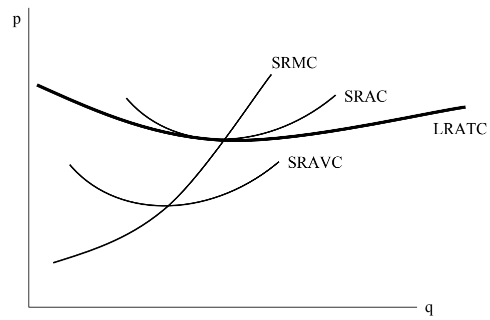
Distribution of Profits with Increasing Returns to Scale

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \ldots x_n \frac{\partial f}{\partial x_n} = \frac{d}{d\lambda} f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n)\Big|_{\lambda \to 1} =$$

$$= \lim_{\lambda \to 1} \frac{f(\lambda x_1, \lambda x_2, \ldots, \lambda x_n) - f(x_1, x_2, \ldots, x_n)}{\lambda - 1} \geq f(x_1, x_2, \ldots, x_n)$$

- Paying inputs their marginal product uses is not generally feasible
- Requires centralization of operations

Firm Costs



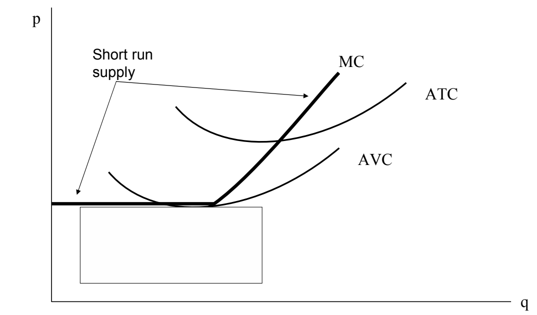
Min AC implies MC=AC

$$0 = \frac{d}{dq}\frac{C(q)}{q} = \frac{C'(q)}{q} - \frac{C(q)}{q^2} \Rightarrow C'(q) = \frac{C(q)}{q}$$

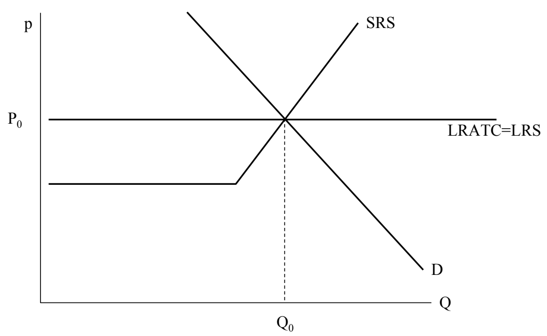
Shut down

- Firm shuts down when price < average cost
- Firm shuts down in short run when price < short run average cost = min average variable cost
- Firm exits in long run when price < long run average cost = min average total cost

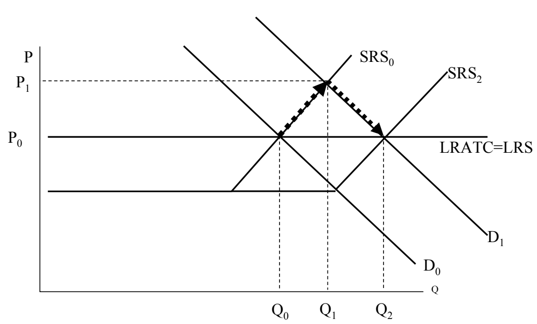
Firm Reaction to Price Changes



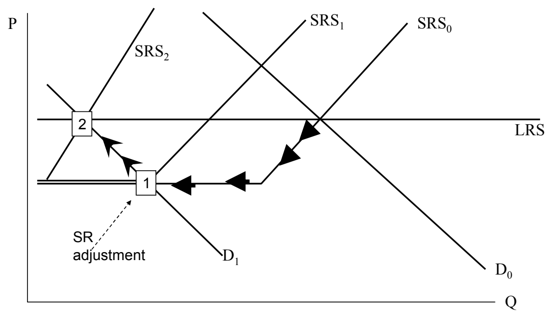
Long-run Equilibrium



Increase in Demand



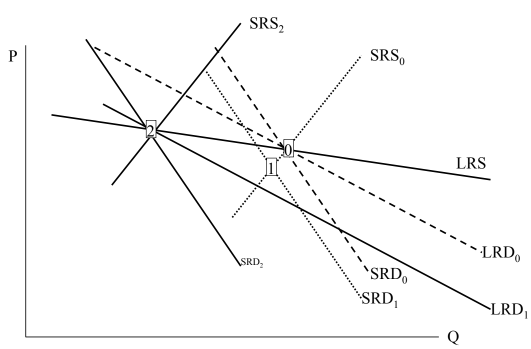
Large Decrease in Demand



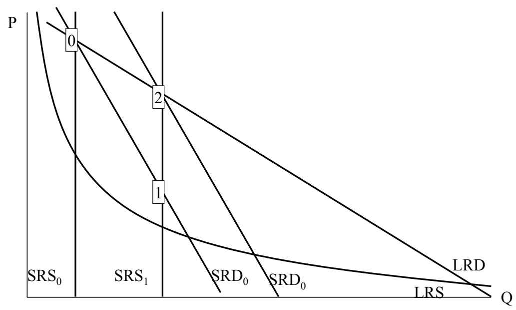
External Economy of Scale

- The size of the industry may affect individual firm costs
  - economy of scale in input supply
  - bidding up price of scarce input
- External economy of scale means LRATC is decreasing in

General Long-run Dynamics



DRAM



Markets

- University Education
- Housing
- Electric cars
- Energy
- Portable music players