Consumer Theory

Budget Set

\[ p_X x + p_Y y \leq M. \]

Isoquants
 Isoquants

- Isoquants are contour sets of the utility function
- Convex preferences means if consumer indifferent between two points, prefers points on the line segment connecting them.

Convex Preferences

Two good Maximization

- Max $u(x,y)$ s.t. $p_x x + p_y y \leq M$
- Max $\frac{M - p_x x}{p_y}

0 = \frac{d}{dx} \left( x \frac{M - p_x x}{p_y} \right) = \frac{\partial u}{\partial x} \frac{p_x}{p_y} + \frac{\partial u}{\partial y} \frac{p_y}{p_y}$.
First Order Condition

\[ 0 = \frac{d}{dx} \left( \frac{M - px}{py} \right) = \frac{\partial u}{\partial x} - \frac{px}{py} \frac{\partial u}{\partial y} \]

\[ \frac{px}{py} \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} = MRS \]

Slope of the budget line = slope of the isoquant

Graphical Illustration

Second Order Condition

• For future reference

\[ 0 \geq \frac{d^2}{(dx)^2} \left( \frac{M - px}{py} \right) = \frac{\partial^2 u}{\partial x^2} - \frac{px}{py} \frac{\partial^2 u}{\partial x \partial y} \left( \frac{px}{py} \frac{\partial^2 u}{\partial y^2} \right) \]
Notation

\[ (u_1, u_2) = \left( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right) \]

- This is the gradient, direction of steepest ascent of \( u \)
- FOC entails gradient perpendicular to budget line

Cobb-Douglas Example

\[ u(x, y) = x^\alpha y^{1-\alpha} \]

\[ \frac{\partial x}{\partial y} = -\frac{\partial y}{\partial x} \bigg|_{u=u_0} = \frac{\partial u/\partial x}{\partial u/\partial y} = \frac{\alpha y}{(1-\alpha)x}. \]

\[ x = \frac{\alpha M}{p_x}, \quad y = \frac{(1-\alpha)M}{p_y} \]

- Constant income shares

Substitution Effect

- Quantity may rise when price goes up
Substitution

• Price increase represents a decrease in purchasing power plus a change in relative price
• Substitution and income effects separate these two

Substitution Effect Holds Utility Constant

Income Effect

• Engel curve traces consumption as income rises
Inferior Good

Decomposition into Substitution and Income Effects

Labor Supply
Increase in wage
• Increases income
  – Increasing leisure, reducing hours
• Increases price of leisure
  – Decreasing leisure
• Labor supplied may decrease as wages rise
U.S. Hours Worked Have Fallen

but wages fell, income grew (two earner households)

Intertemporal Choice

- \( u(x_1, x_2) = v(x_1) + \delta v(x_2) \)
- Budget \((1+r)x_1 + x_2 = (1+r)M_1 + M_2\)
- FOC \(0 = v'(x_1) - (1+r)\delta v'(x_2)\)

Intertemporal Optimization
Interest Rate Increase

Borrower’s Income Falls

\(M_1, M_2\)

\(x_1\)

\(x_2\)

Interest Rate Increase

Creditor’s Income Rises

\(M_1, M_2\)

\(x_1\)

\(x_2\)

Neither Borrower Nor Lender Be

Differences in Borrowing/Lending Rates

\(M_1, M_2\)

\(x_1\)

\(x_2\)
Risk

- Von Neumann-Morgenstern Utility
- Value gambles at expected utility

\[ u = \pi_1 v(x_1) + \pi_2 v(x_2) + \ldots + \pi_n v(x_n) = \sum_{i=1}^{n} \pi_i v(x_i) \]

Risk Aversion

- Risk averse if prefer expected outcome

\[ v(\pi x_1 + (1-\pi) x_2) \geq \pi v(x_1) + (1-\pi) v(x_2) \]

- Risk averse to all gambles if and only if concave utility (second derivative < 0)
Definitions

- Certainty Equivalent is the amount of money valued the same as the gamble is valued
- Risk premium is the expected value of gamble minus the certainty equivalent
- Risk premium is the cost of risk

Measuring Risk Premium

- For small gambles
  \[ \nu(\mu) + \nu'(\mu)(CE - \mu) = \nu(CE) = E(\nu(x)) = \approx E(\nu(\mu) + \nu'(\mu)(x - \mu) + \frac{1}{2} \nu''(\mu)(x - \mu)^2) \]
  \[ \mu - CE \approx -\frac{1}{2} \frac{\nu''(\mu)}{\nu'(\mu)} \sigma^2 \]
  \[ -\frac{\nu''(\mu)}{\nu'(\mu)} \] is the Arrow-Pratt measure of constant absolute risk aversion

Absolute Risk Aversion

- Absolute risk aversion generally thought to be decreasing in wealth
- If so, an increase in wealth reduces the risk premium for any gamble
Means-Variance

Constant absolute risk aversion \( \rho = \frac{\nu'(x)}{\nu(x)} \)
plus normally distributed gambles
implies certainty equivalent of
\[ CE = \mu - \frac{1}{2}\rho \sigma^2 \]
Such preferences give linear payoff to mean
and variance
Used in finance, agency theory

Search

- Consumers face random price, density \( f \)
- Cost of obtaining price quote \( c \)
- Use reservation price, buy if \( p \leq p^* \)
- Expected cost

\[
J(p^*) = \int_0^{p^*} pf(p)dp + \int_{p^*}^{\infty} (J(p^*) + c)f(p)dp
\]

Expected Cost of Purchase

\[
J(p^*) = \int_0^{p^*} \frac{pf(p)dp + c}{F(p^*)} = \frac{f(p^*)}{F(p^*)} \int_0^{p^*} \frac{pf(p)dp + c}{F(p^*)^2}
\]

- FOC:

\[
J'(p^*) = p^* f(p^*) \frac{F(p^*)}{F(p^*)} - \frac{f(p^*)}{F(p^*)} \int_0^{p^*} \frac{pf(p)dp + c}{F(p^*)^2}
\]

\[
= \frac{f(p^*)}{F(p^*)} \left[ p^* - \int_0^{p^*} \frac{pf(p)dp + c}{F(p^*)} \right] = \frac{f(p^*)}{F(p^*)} (p^* - J(p^*))
\]
Solution

- Unique \( J(p^*) = p^* \)
- Buy if and only if price offered is less than the expected cost of future purchase

Edgeworth Box

- Two people, two goods
- Given total endowment of goods

Pareto Efficiency
General Equilibrium

- \( n \) goods, \( I \) people, convex preferences
- First welfare theorem: any price system equilibrium is pareto efficient
- Second welfare theorem: any efficient point is a price system equilibrium for some endowment
- There exists a price system equilibrium