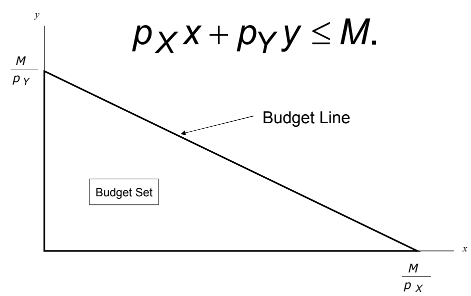
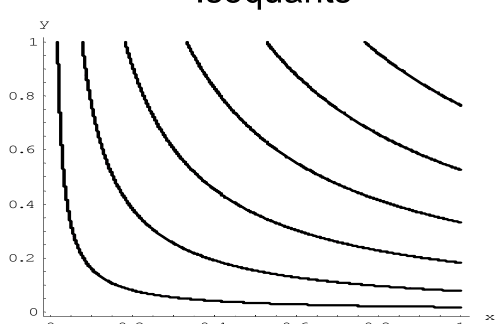
## Consumer Theory

## Budget Set

$$p_X x + p_Y y \leq M.$$



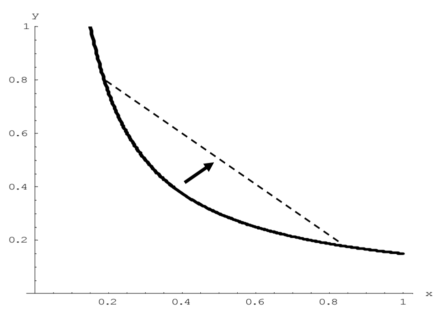

## Isoquants



## Isoquants

- Isoquants are contour sets of the utility function
- Convex preferences means if consumer indifferent between two points, prefers points on the line segment connecting them.

## Convex Preferences




## Two good Maximization

- Max $u(x,y)$ s.t. $p_X x + p_Y y \le M$
- Max $u\left(x, \frac{M - p_X x}{p_Y}\right).$

$$0 = \frac{d}{dx} u\left(x, \frac{M - p_X x}{p_Y}\right) = \frac{\partial u}{\partial x} - \frac{p_X}{p_Y}\frac{\partial u}{\partial y}.$$

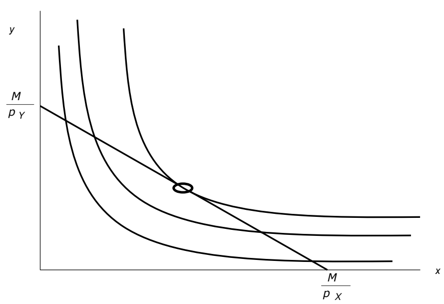
## First Order Condition

$$0 = \frac{d}{dx} u\left(x, \frac{M - p_X x}{p_Y}\right) = \frac{\partial u}{\partial x} - \frac{p_X}{p_Y}\frac{\partial u}{\partial y}.$$

$$\frac{p_X}{p_Y} = \frac{\partial u / \partial x}{\partial u / \partial y} = -\left.\frac{dy}{dx}\right|_{u=u_0} = MRS$$

Slope of the budget line = slope of the isoquant

## Graphical Illustration




## Second Order Condition

- For future reference

$$0 \geq \frac{d^2}{(dx)^2} u\left(x, \frac{M - p_X x}{p_Y}\right) = \frac{\partial^2 u}{(\partial x)^2} - \frac{p_X}{p_Y}\frac{\partial^2 u}{\partial x \partial y} + \left(\frac{p_X}{p_Y}\right)^2 \frac{\partial^2 u}{(\partial y)^2}$$

## Notation

$$(u_1, u_2) = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right)$$

- This is the gradient, direction of steepest ascent of *u*
- FOC entails gradient perpendicular to budget line

## Cobb-Douglas Example

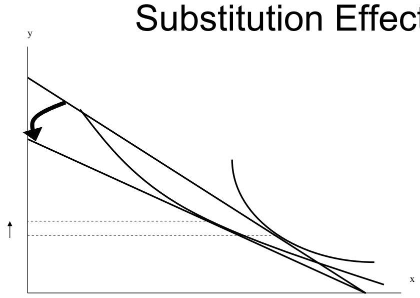
$$u(x,y) = x^{\alpha} y^{1-\alpha}$$

$$\frac{p_X}{p_Y} = -\left.\frac{dy}{dx}\right|_{u=u_0} = \frac{\partial u / \partial x}{\partial u / \partial y} = \frac{\alpha y}{(1-\alpha)x}.$$

$$x = \frac{\alpha M}{p_X}, \quad y = \frac{(1-\alpha)M}{p_Y}$$

- Constant income shares

## Substitution Effect

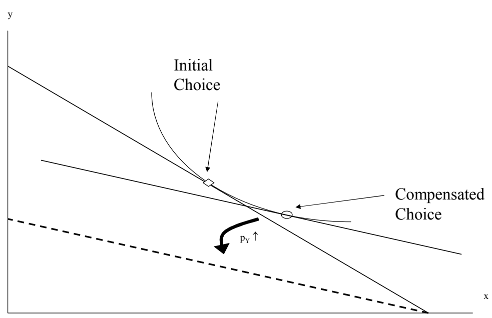



- Quantity may rise when price goes up

## Substitution

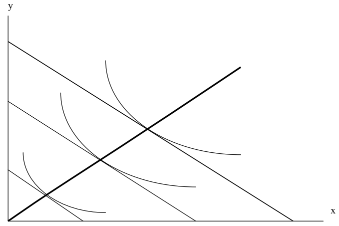
- Price increase represents a decrease in purchasing power plus a change in relative price
- Substitution and income effects separate these two

## Substitution Effect Holds Utility Constant

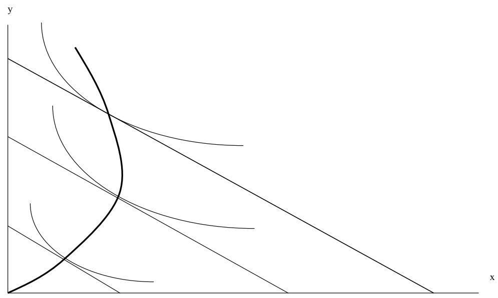



## Income Effect

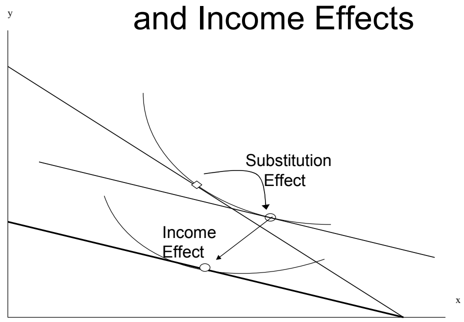
- Engel curve traces consumption as income rises



## Inferior Good




## Decomposition into Substitution and Income Effects

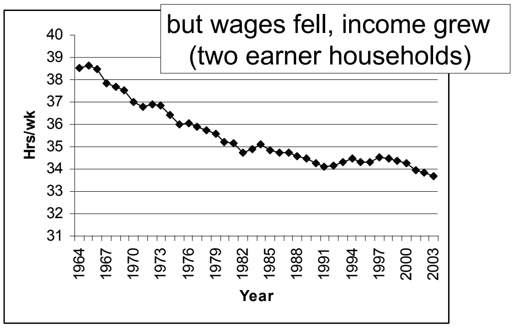



## Labor Supply

Increase in wage

- Increases income
  - Increasing leisure, reducing hours
- Increases price of leisure
  - Decreasing leisure
- Labor supplied may decrease as wages rise

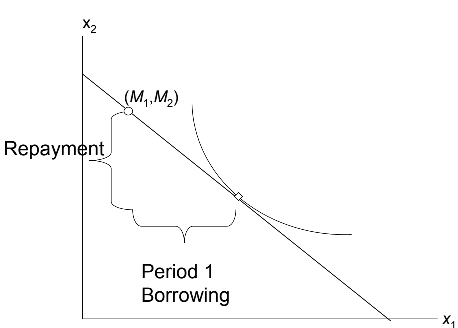
## U.S. Hours Worked Have Fallen



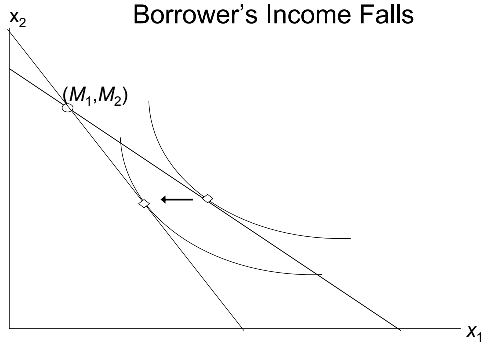

## Intertemporal Choice

- $u(x_1, x_2) = v(x_1) + \delta v(x_2)$
- Budget $(1+r)x_1 + x_2 = (1+r)M_1 + M_2$
- FOC $0 = v'(x_1) - (1+r)\delta v'(x_2)$

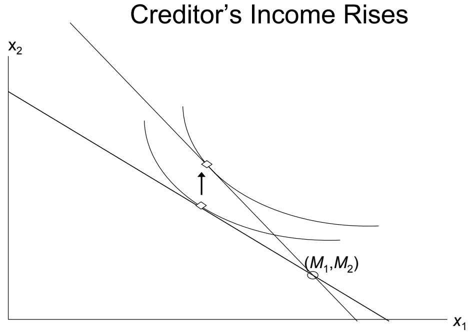
## Intertemporal Optimization



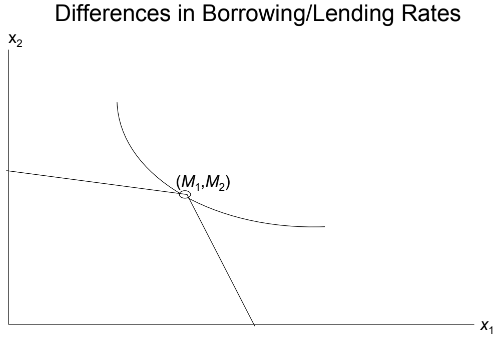
## Interest Rate Increase
Borrower's Income Falls




## Interest Rate Increase
Creditor's Income Rises




## Neither Borrower Nor Lender Be
Differences in Borrowing/Lending Rates



## Risk

- Von Neumann-Morgenstern Utility
- Value gambles at expected utility

$$u = \pi_1 v(x_1) + \pi_2 v(x_2) + \ldots + \pi_n v(x_n) = \sum_{i=1}^{n} \pi_i v(x_i)$$

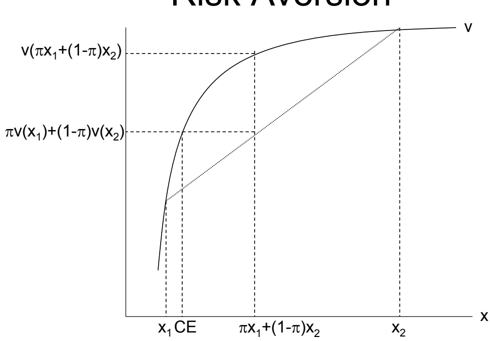
## Risk Aversion

- Risk averse if prefer expected outcome

$$v(\pi x_1 + (1-\pi) x_2) \geq \pi v(x_1) + (1-\pi) v(x_2)$$

- Risk averse to all gambles if an only if concave utility (second derivative < 0)

## Risk Aversion



## Definitions

- Certainty Equivalent is the amount of money valued the same as the gamble is valued
- Risk premium is the expected value of gamble minus the certainty equivalent
- Risk premium is the cost of risk

## Measuring Risk Premium

- For small gambles

$$v(\mu) + v'(\mu)(CE - \mu) \approx v(CE) = E\{v(x)\} \approx$$

$$\approx E\{v(\mu) + v'(\mu)(x - \mu) + ½v''(\mu)(x - \mu)^2\}$$

$$\mu - CE \approx -½\frac{v''(\mu)}{v'(\mu)}\sigma^2$$

$-\frac{v''(\mu)}{v'(\mu)}$ is the Arrow Pratt measure of constant absolute risk aversion

## Absolute Risk Aversion

- Absolute risk aversion generally thought to be decreasing in wealth
- If so, an increase in wealth reduces the risk premium for any gamble

## Means-Variance

Constant absolute risk aversion plus normally distributed gambles implies certainty equivalent of

$$\rho = -\frac{v''(x)}{v'(x)}$$

$$CE = \mu - ½\rho\sigma^2$$

Such preferences give linear payoff to mean and variance

Used in finance, agency theory

## Search

- Consumers face random price, density *f*
- Cost of obtaining price quote *c*
- Use reservation price, buy if $p \leq p^*$
- Expected cost

$$J(p^*) = \int_0^{p^*} pf(p)dp + \int_{p^*}^{\infty} (J(p^*) + c)f(p)dp$$

## Expected Cost of Purchase

$$J(p^*) = \frac{\int_0^{p^*} pf(p)dp + c}{F(p^*)}.$$

- FOC: $J'(p^*) = p^* \frac{f(p^*)}{F(p^*)} - \frac{f(p^*)\int_0^{p^*} pf(p)dp + c}{F(p^*)^2}$

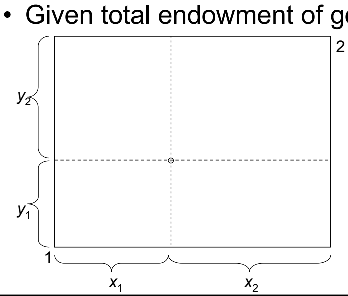
$$= \frac{f(p^*)}{F(p^*)}\left(p^* - \frac{\int_0^{p^*} pf(p)dp + c}{F(p^*)}\right) = \frac{f(p^*)}{F(p^*)}\left(p^* - J(p^*)\right).$$

## Solution

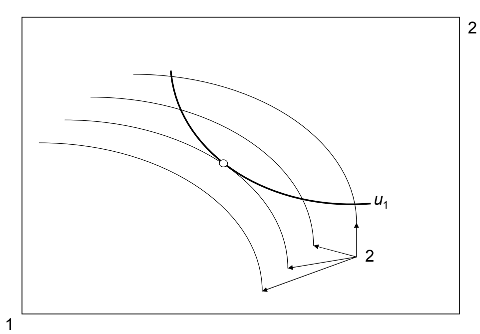
- Unique $J(p^*)=p^*$
- Buy if and only if price offered is less than the expected cost of future purchase

## Edgeworth Box

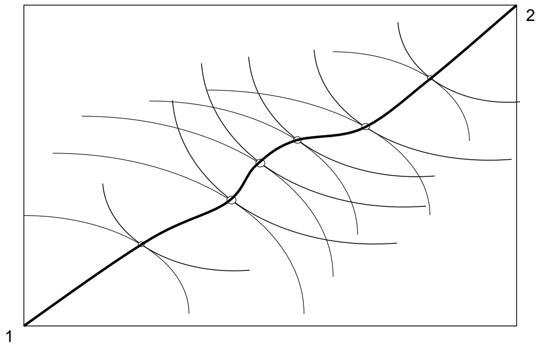
- Two people, two goods
- Given total endowment of goods



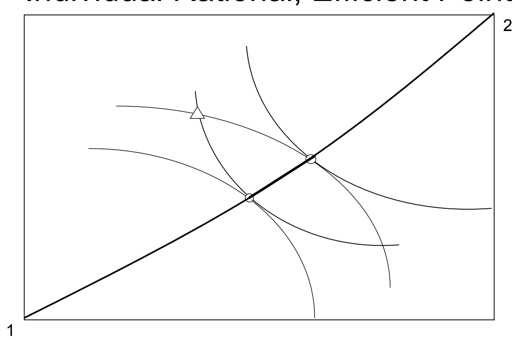

## Pareto Efficiency



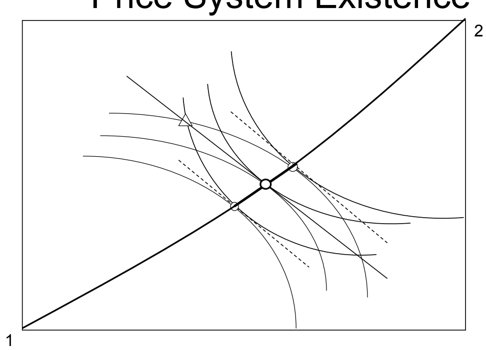
## Contract Curve




## Individual Rational, Efficient Points




## Price System Existence



## General Equilibrium

- *n* goods, *I* people, convex preferences
- First welfare theorem: any price system equilibrium is pareto efficient
- Second welfare theorem: any efficient point is a price system equilibrium for some endowment
- There exists a price system equilibrium