Ec 11 Homework 3
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CALTECH

1. Consider a cost function of producing an output q of the form $c(q) = q^2+2q+16$. Determine:
    a. Marginal cost
    b. Average cost
    c. Average variable cost
    d. Graph the long run supply curve assuming the cost function is for a single plant, and can be replicated without change.
2. Consider two consumers and two goods, X and Y. Consumer 1 has utility $u_1(x_1, y_1) = x_1 + y_1$ and Consumer 2 has utility $u_2(x_2, y_2) = \min\{x_2, y_2\}$. Consumer 1 has an endowment of (1, 1/2) and Consumer 2's endowment is (0, 1/2).
    a. Draw the Edgeworth box for this economy.
    b. Find the contract curve, and the individually rational part of it. (You should describe these in writing and highlight them in the Edgeworth box.)
    c. Find the prices that support an equilibrium of the system, and the final allocation of goods under those prices.

For questions 3-6 consider an orange juice factory that uses as inputs oranges and workers. If the factory uses x pounds of oranges and y workers per hour, it produces

$$T = 20\, x^{0.25}\, y^{0.5}$$

gallons of orange juice.

3. Suppose oranges cost $1 and workers cost $10, what relative proportion of oranges and workers should the factory use?
4. Suppose a gallon of orange juice sells for $1, how many units should be sold and what is the input mix to be used? What is the profit?
5. Generalize the previous exercise for a price of $*p* per gallon of orange juice.
6. What is the supply elasticity?

7. For experiment 2.1, draw the demand, short-run supply (given actual number of restaurants) and long-run supply. [Hint: How many restauranteurs would open restaurants if the price is *11 (that is, they can sell all they want at a price of *11)? At a price of *9?]. Find short and long run equilibrium prices and quantities.
8. (short answer) Why would a restaurant be willing to sell for a price less than its average total cost?
9. Over the course of the experiments, did entry respond to profits?

1. Consider a cost function of producing an output q of the form $c(q) = q^2+2q+16$. Determine:
    a. Marginal cost

$MC(q) = c'(q) = q+2$

    b. Average cost

$AC(q) = \frac{q^2+2q+16}{q}$

    c. Average variable cost

$AVC(q) = q+2$

    d. Graph the long run supply curve assuming the cost function is for a single plant, and can be replicated without change.

Average cost is $\frac{q^2+2q+16}{q}$, which is minimized when $q = 4$. This corresponds to an average cost of 10, so long run supply is a horizontal at an average cost of 10.

2. Consider two consumers and two goods, X and Y. Consumer 1 has utility $u_1(x_1, y_1) = x_1 + y_1$ and Consumer 2 has utility $u_2(x_2, y_2) = \min\{x_2, y_2\}$. Consumer 1 has an endowment of (1, 1/2) and Consumer 2's endowment is (0, 1/2).
    a. Draw the Edgeworth box for this economy.

Picture should be of a 1x1 box. Consumer 1's indifference curves have slope –1, and consumer 2's indifference curves are L-shaped with vertices along 45-degree line.

    b. Find the contract curve, and the individually rational part of it. (You should describe these in writing and highlight them in the Edgeworth box.)

The contract curve is simply the 45-degree line. The individually rational segment is where $1 >= x_1 >= 3/4$.

    c. Find the prices that support an equilibrium of the system, and the final allocation of goods under those prices.

It is easiest to find the price ratio graphically, by drawing a straight line from the endowment such that it is tangent to the indifference curves of both consumers at the contract curve. This line will have slope –1, so the price ratio of X to Y is 1. At this price, consumer 1 ends up with (3/4, 3/4) and 2 gets (1/4, 1/4).

For questions 3-6 consider an orange juice factory that uses as inputs oranges and workers. If the factory uses x pounds of oranges and y workers per hour, it produces

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gallons of orange juice.

3. Suppose oranges cost $1 and workers cost $10, what relative proportion of oranges and workers should the factory use?

First solve for $y = \frac{T^2}{400\sqrt{x}}$, giving an expression for cost, $C = x + 10y = x + \frac{T^2}{40\sqrt{x}}$. We differentiate C to find the minimizing x to be $x = \frac{T^{4/3}}{4 \cdot 10^{2/3}}$. Then $\frac{x}{y} = \frac{x}{T^2 / 400\sqrt{x}} = 5$

4. Suppose a gallon of orange juice sells for $1, how many units should be sold and what is the input mix to be used? What is the profit?

See below for the general case. When $p = 1$, we have $T = 100$, $x = 25$, profit = 25.

5. Generalize the previous exercise for a price of $$p$ per gallon of orange juice.

From previous answer we know $y = 1/5\, x$, so profit is given by
$pT - x - 10y = pT - 3x = pT - 3\frac{T^{4/3}}{4 \cdot 10^{2/3}}$. Differentiating this and finding the maximizing T we find that $T = 100p^3$ and, upon substitution of our relation between T and x found earlier, we have $x = 25p^4$. Using these values of T, p,
profit becomes $pT - x - 10y = pT - 3x = 100p^4 - 75p^4 = 25p^4$.

6. What is the supply elasticity?

$$\eta = \frac{pq'(p)}{q(p)} = \frac{3 \cdot 100p^3}{100p^3} = 3$$