



Instructions: Open book, open notes, no collaboration.
Partial credit will be assigned. Please show your work.
You may take this test during any consecutive 4 hour period.

Due February 7, by 5:00 PM. Please deposit in Box outside Baxter 100.

- The demand for hamburgers has a constant elasticity of 1 of the form $x(p) = 8,000 p^{-1}$. Each entrant in this competitive industry has a fixed cost of \$2,000 and produces \sqrt{x} hamburgers per year, where x is the amount of meat in pounds.
 - If the price of meat is \$2/lbs, what is the long run supply of hamburgers?
 - Compute the equilibrium number of firms, quantity supplied by each firm and the market price of hamburgers.
 - Find the short run industry supply, does it have constant elasticity?
- A recent college graduate wants to retire 30 years from today, and would like to spend \$20,000 during the first year of his retirement, in today's terms. Suppose his annual retirement expenditure increases 3% every year. If he lives forever, how much should he save each year leading up to his retirement? Assume a 5% return on investments.
- A toy factory costs \$2 million to construct and the marginal cost of the q^{th} toy is $\text{Max}[10, q^2/1,000]$.
 - What are average total costs?
 - What is short run supply?
 - What is the long run competitive supply of toys?
- Give a brief summary of Ricardian theory. If it holds true, what kinds of goods should the US export and what should it import? How well does the theory hold up?
- A company that produces software needs two inputs, programmers (x) at a rate of p and computers (y) at a price of r . The output is given by $T = 4 x^{1/3} y^{1/3}$, measured in pages of code.
 - What is marginal cost?
 - Now suppose each programmer needs two computers to do his job, what ratio of p and r would make this input mix optimal?
- Suppose the price of Microsoft's operating system increases. Explain how this might affect the price and quantity of Intel microprocessor chips.

Midterm Answers:

1a) Let q be the total number of hamburgers produced, then Average Cost = $(2000+2q^2)/q$. This is minimized when $q = 10\sqrt{10}$, with an average cost of $40\sqrt{10}$, which is the price of the long run supply.

1b) At a price of $40\sqrt{10}$, the quantity demanded is $x(p) = 8,000(40\sqrt{10})^{-1} = 20\sqrt{10}$. This represents 2 firms, each supplying $10\sqrt{10}$.

1c) Short run supply is marginal cost, which is $4q$, so $P = 4q/N$, where N is the number of firms. Yes, it is in the form of constant elasticity.

2) The present value of the retirement costs is:

$$PV = \frac{20,000}{(1.05)^{30}} + \frac{20,000(1.03)}{(1.05)^{31}} + \frac{20,000(1.03)^2}{(1.05)^{32}} + \dots = \frac{20,000}{(1.05)^{30}} \left[\frac{1}{1 - \left(\frac{1.03}{1.05}\right)} \right] =$$

\$242,946.32

The present value of the retirement payments (call the amount A) is:

$$PV = \frac{A}{1.05} + \frac{A}{(1.05)^2} + \dots + \frac{A}{(1.05)^{30}} = 15.37 * A = 242,946.32$$

Solving for A , the annual payment is $A = 15,804.01$

Alternatively, you could have made the retirement payments at the beginning of each year, and the series is then $PV = A + \frac{A}{(1.05)^1} + \dots + \frac{A}{(1.05)^{29}} = 16.14 A$, which implies that $A = 15,051.44$.

Another alternative would have been to say that the PV of retirement costs is

$$\frac{20,000}{(1.05)^{30}} \left[1 + \frac{1}{(0.05 - 0.03)} \right] = 236,005, \text{ and solve for } A \text{ accordingly.}$$

Any answer in which A was determined with a set of these reasonable assumptions is acceptable.

3a) Average total costs are:

If $q < 100$, $ATC = 2,000,000/q + 10$

If $q \geq 100$, $ATC = (2,001,000 - 1000/3)/q + q^2/3000$

3b) Short run supply is the marginal cost:

If $q < 100$, $MC = 10$

If $q \geq 100$, $MC = q^2/1000$

3c) To find long run supply we find the minimum average cost. Since it is decreasing for $q < 100$, we minimize $(2,001,000 - 1000/3)/q + q^2/3000$ which solves for $q = 1442.41$, with an average cost of 2080.55, which is the price in the long run.

4) Ricardian theory states that nations should produce and trade a good in which it has a comparative advantage. Since the US has an abundance of land and highly skilled labor, it should produce goods with those factors as inputs. Major exports include semiconductors, movies, and grain. Major imports include oil, home electronics, and cars. The theory holds up fairly well for predicting the US's imports and exports. However, it also predicts more trade between developed and developing countries than actually exists.

5a) Total cost is given by $px + r\left(\frac{T}{4x^{1/3}}\right)^3$, which when minimized leads to

$$x = \sqrt{\frac{r}{p}} \frac{T^{3/2}}{8} \text{ and } y = \sqrt{\frac{p}{r}} \frac{T^{3/2}}{8} \text{ and so } C = \sqrt{pr} \frac{T^{3/2}}{4} \text{ and } MC = D_r[C] = \frac{3}{8} \sqrt{prT}$$

5b) From above, $x = \sqrt{\frac{r}{p}} \frac{T^{3/2}}{8}$ and $y = \sqrt{\frac{p}{r}} \frac{T^{3/2}}{8}$ for minimized cost and so $x/y = r/p$. Since $2x = y$, $r/p = 1/2$.

6) Microsoft operating systems and Intel microprocessors are complements in demand. Hence when the price of operating systems rises, demand for microprocessors decrease and so price and quantity fall.