

Ec 11 Homework 8
Professor R. Preston McAfee
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CALTECH



1. Suppose the inverse demand curve is $p(Q) = 1 - Q$, and that there are n Cournot firms, each with marginal cost c selling in the market.
 - a. Find the Cournot equilibrium price and quantity
 - b. Determine the gross profits for each firm
2. What formula from the Cournot model is used in antitrust analysis? How is it used?
3. Consider n identical Cournot firms in equilibrium.
 - a. Show that the elasticity of market demand satisfies $\varepsilon > 1/n$.
 - b. Is this consistent in the case when $n = 1$ (monopoly)?
4. Describe how a principal would go about hiring agents who are willing to take risks.

1. Suppose the inverse demand curve is $p(Q) = 1 - Q$, and that there are n Cournot firms, each with marginal cost c selling in the market.
 - a. Find the Cournot equilibrium price and quantity

We take q^* to be each firm's output and maximize the profits as a function of quantity q (after we find the first order condition we set $q=q^*$ by symmetry):

$$\pi = p(Q)q - c(q) = (1 - (n-1)q^* - q)q - cq$$

First order condition -

$$1 - (n-1)q^* - 2q - c = 0$$

$$\Rightarrow 1 - (n+1)q - c = 0$$

$$\Rightarrow q = \frac{1-c}{n+1}$$

$$\Rightarrow Q = nq = \frac{n(1-c)}{n+1}$$

Then price is given by:

$$p(Q) = 1 - Q = \frac{1+nc}{n+1}$$

- b. Determine the gross profits for each firm

Profits are given by

$$\pi = p(Q)q - c(q) = (p(Q) - c)q = \left(\frac{1+nc}{n+1} - c\right)\frac{1-c}{n+1} = \left(\frac{1-c}{n+1}\right)^2$$

2. What formula from the Cournot model is used in antitrust analysis? How is it used?

The HHI is used in assessing the effects of increased industry concentration that would arise from a merger.

3. Consider n identical Cournot firms in equilibrium.
 - a. Show that the elasticity of market demand satisfies $\varepsilon > 1/n$.

The first order condition can be written as,

$$0 = p(Q) + p'(Q)\frac{Q}{n} - c'\left(\frac{Q}{n}\right)$$

$$\Rightarrow \frac{p(Q) - c'\left(\frac{Q}{n}\right)}{p(Q)} = -\frac{p'(Q)Q}{p(Q)} \frac{1}{n} = -\frac{dP(Q)/P(Q)}{dQ/Q} = \frac{1}{\varepsilon n}$$

Since the marginal cost is nonnegative, we must have

$$\frac{1}{\varepsilon n} < 1 \Rightarrow \varepsilon > \frac{1}{n}$$

b. Is this consistent in the case when $n = 1$ (monopoly)?

For a monopoly, $\varepsilon > 1$, which is consistent with part a.

4. Describe how a principal would go about hiring agents who are willing to take risks.

The principal should offer a contract with a low salary but high commission.